Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Α

18.4 1)

α) $$\lim_{x\to0^+}\frac{\ln(\text{ημ}x)}{\ln(\text{εφ}x)}\overset{\frac{+\infty}{+\infty}}{\underset{DLH}{=}}\lim_{x\to0^+}\frac{\frac{1}{\text{ημ}x}\cdot\text{συν}x}{\frac{1}{\text{εφ}x}\cdot\frac{1}{\text{συν}^2x}}=\lim_{x\to0^+}\frac{\frac{1}{\cancel{\text{ημ}x}}\cdot\text{συν}x}{\frac{\cancel{\text{συν}x}}{\cancel{\text{ημ}x}}\cdot\frac{1}{\text{συν}^{\cancel{2}}x}}=\lim_{x\to0^+}\frac{\text{συν}x}{\frac{1}{\text{συν}x}}=\frac{1}{1}=1$$

β) $$\lim_{x\to1}\frac{x-1-\ln x}{(3x^3-2)(x-1)\ln x}=\lim_{x\to1}\frac{1}{3x^3-2}\cdot\lim_{x\to1}\frac{x-1-\ln x}{(x-1)\ln x}=$$

$$=\frac{1}{3\cdot1^3-2}\cdot\lim_{x\to1}\frac{x-1-\ln x}{(x-1)\ln x}=1\cdot\lim_{x\to1}\frac{x-1-\ln x}{(x-1)\ln x}\overset{\frac{0}{0}}{\underset{DLH}{=}}\lim_{x\to1}\frac{1-\frac{1}{x}}{\ln x+\frac{x-1}{x}}=$$

$$=\lim_{x\to1}\frac{\frac{x-1}{\cancel{x}}}{\frac{x\ln x+x-1}{\cancel{x}}}=\lim_{x\to1}\frac{x-1}{x\ln x+x-1}\overset{\frac{0}{0}}{\underset{DLH}{=}}\lim_{x\to1}\frac{1}{\ln x+\cancel{x}\frac{1}{\cancel{x}}+1}=\frac{1}{2}$$

18.4 2)

$$\boxed{\lim_{x\to+\infty}\frac{e^{\frac{1}{x}}\ln^2x}{x+2}}=\lim_{x\to+\infty}e^{\frac{1}{x}}\cdot\lim_{x\to+\infty}\frac{\ln^2x}{x+2}\overset{\lim_{x\to+\infty}\frac{1}{x}=0}{=}e^0\cdot\lim_{x\to+\infty}\frac{\ln^2x}{x+2}\overset{\frac{+\infty}{+\infty}}{\underset{DLH}{=}}\lim_{x\to+\infty}\frac{2\ln x\cdot\frac{1}{x}}{1}=$$

$$=\lim_{x\to+\infty}\frac{2\ln x}{x}\overset{\frac{+\infty}{+\infty}}{\underset{DLH}{=}}\lim_{x\to+\infty}\frac{2\frac{1}{x}}{1}=\frac{2}{+\infty}=\boxed{0}$$

18.4 3)

$$\boxed{\lim_{x\to+\infty}\frac{x^2+\ln2x}{e^{2x}\text{συν}\frac{1}{x}}}=\lim_{x\to+\infty}\frac{1}{\text{συν}\frac{1}{x}}\cdot\lim_{x\to+\infty}\frac{x^2+\ln2x}{e^{2x}}\overset{\lim_{x\to+\infty}\frac{1}{x}=0}{=}\frac{1}{\text{συν}0}\cdot\lim_{x\to+\infty}\frac{x^2+\ln2x}{e^{2x}}\overset{\frac{+\infty}{+\infty}}{\underset{DLH}{=}}$$

$$=\lim_{x\to+\infty}\frac{2x+\frac{1}{\cancel{2}x}\cdot\cancel{2}}{e^{2x}\cdot2}=\lim_{x\to+\infty}\frac{\frac{2x^2+1}{x}}{e^{2x}\cdot2}=\lim_{x\to+\infty}\frac{2x^2+1}{2xe^{2x}}\overset{\frac{+\infty}{+\infty}}{\underset{DLH}{=}}\lim_{x\to+\infty}\frac{4x}{2e^{2x}+4xe^{2x}}\overset{\frac{+\infty}{+\infty}}{\underset{DLH}{=}}$$

$$=\frac{4}{+\infty}=\boxed{0}$$

18.4 4)

$$\boxed{\lim_{x\to0}\frac{e^{x^2}-1}{(2-2\text{συν}x)2^x}}=\lim_{x\to0}2^x\cdot\lim_{x\to0}\frac{e^{x^2}-1}{2-2\text{συν}x}=2^0\cdot\lim_{x\to0}\frac{e^{x^2}-1}{2-2\text{συν}x}=$$

$$=\lim_{x\to0}\frac{e^{x^2}-1}{2-2\text{συν}x}\overset{\frac{0}{0}}{\underset{DLH}{=}}\lim_{x\to0}\frac{e^{x^2}\cdot\cancel{2}x}{\cancel{2}\text{ημ}x}=\lim_{x\to0}e^{x^2}\cdot\lim_{x\to0}\frac{x}{\text{ημ}x}\overset{\lim_{x\to0}\frac{x}{\text{ημ}x}=1}{=}e^0\cdot1=\boxed{1}$$

18.4 5)

$$\lim_{x\to0^+}\frac{\ln(\text{ημ}x)}{\ln(1-\text{συν}x)}\overset{\frac{+\infty}{+\infty}}{\underset{DLH}{=}}\lim_{x\to0^+}\frac{[\ln(\text{ημ}x)]'}{[\ln(1-\text{συν}x)]'}=\lim_{x\to0^+}\frac{\frac{1}{\text{ημ}x}\text{συν}x}{\frac{1}{1-\text{συν}x}\text{ημ}x}=$$

$$= \lim_{x\to 0^+}\frac{\frac{1}{ημx}συνx}{\frac{1}{1-συνx}ημx} = \lim_{x\to 0^+}συνx\frac{1-συνx}{ημ^2x} = \lim_{x\to 0^+}συνx\cdot\lim_{x\to 0^+}\frac{1-συνx}{ημ^2x}$$

$$=1\cdot\lim_{x\to 0^+}\frac{1-συνx}{ημ^2x}\underset{DLH}{\overset{\frac{0}{0}}{=}}\lim_{x\to 0^+}\frac{(1-συνx)'}{(ημ^2x)'} = \lim_{x\to 0^+}\frac{\overset{1}{\cancel{ημx}}}{2\,\cancel{ημx}\,συνx} = \frac{1}{2}$$

18.4 6)

$$\boxed{\lim_{x\to 0^+}\frac{\frac{3π}{2x}}{σφ\frac{πx}{2}}} = \lim_{x\to 0^+}\frac{\frac{3π}{2x}}{\frac{συν\frac{πx}{2}}{ημ\frac{πx}{2}}} = \lim_{x\to 0^+}\frac{3πημ\frac{πx}{2}}{2xσυν\frac{πx}{2}} = \lim_{x\to 0^+}\frac{3π}{2συν\frac{πx}{2}}\cdot\lim_{x\to 0^+}\frac{ημ\frac{πx}{2}}{x} =$$

$$=\frac{3π}{2}\cdot\lim_{x\to 0^+}\frac{ημ\frac{πx}{2}}{x}\underset{DLH}{\overset{\frac{0}{0}}{=}}\frac{3π}{2}\cdot\lim_{x\to 0^+}\frac{\frac{π}{2}συν\frac{πx}{2}}{1} = \frac{3π}{2}\cdot\frac{π}{2} = \boxed{\frac{3π^2}{4}}$$

18.4 7)

$$\lim_{x\to 0}\frac{συνx(συνx-1)}{(1-e^x)ημx} = \lim_{x\to 0}συνx\cdot\lim_{x\to 0}\frac{συνx-1}{(1-e^x)ημx} = συν0\cdot\lim_{x\to 0}\frac{συνx-1}{(1-e^x)ημx} =$$

$$=1\cdot\lim_{x\to 0}\frac{συνx-1}{(1-e^x)ημx} = \lim_{x\to 0}\frac{συνx-1}{(1-e^x)ημx}\underset{DLH}{\overset{\frac{0}{0}}{=}}\lim_{x\to 0}\frac{-ημx}{-e^xημx+(1-e^x)συνx}\underset{DLH}{\overset{\frac{0}{0}}{=}}$$

$$=\lim_{x\to 0}\frac{-συνx}{-e^xημx-e^xσυνx-e^xσυνx-(1-e^x)ημx} =$$

$$=\frac{-συν0}{-e^0ημ0-e^0συν0-e^0συν0-(1-e^0)ημ0} = \frac{-1}{-2} = \boxed{\frac{1}{2}}$$

18.4 8)

$$\boxed{\lim_{x\to 0^+}\frac{\ln 2x}{σφ3x}}\underset{DLH}{\overset{\frac{+\infty}{+\infty}}{=}}\lim_{x\to 0^+}\frac{\frac{1}{\cancel{2}x}\cdot\cancel{2}}{-\frac{1}{ημ^2 3x}\cdot 3} = \lim_{x\to 0^+}\frac{-ημ^2 3x}{3x} = \lim_{x\to 0^+}(-ημ3x)\cdot\lim_{x\to 0^+}\frac{ημ3x}{3x} = 0\cdot 1 = \boxed{0}$$

18.4 9)

$$\boxed{\lim_{x\to 2^-}\frac{\ln(2-x)}{εφ\frac{πx}{4}}}\underset{DLH}{\overset{\frac{+\infty}{+\infty}}{=}}\lim_{x\to 2^-}\frac{\frac{-1}{2-x}}{\frac{1}{συν^2\frac{πx}{4}}\frac{π}{4}} = \lim_{x\to 2^-}\frac{-4συν^2\frac{πx}{4}}{π(2-x)}\underset{DLH}{\overset{\frac{0}{0}}{=}}$$

$$=\lim_{x\to 2^-}\frac{-\cancel{4}\cdot 2\cdot συν\frac{πx}{4}\left(-ημ\frac{πx}{4}\right)\cdot\frac{π}{\cancel{4}}}{-π} = \lim_{x\to 2^-}\frac{2\cdot\cancel{π}συν\frac{πx}{4}ημ\frac{πx}{4}}{-\cancel{π}} =$$

$$=-2συν\frac{π}{2}ημ\frac{π}{2} = -2\cdot 0\cdot 1 = \boxed{0}$$

18.4 10)

$$\boxed{\lim_{x\to0}\frac{2x-\varepsilon\varphi 2x}{1-e^x-\ln(1-x)}}\overset{\frac{0}{0}}{\underset{DLH}{=}}\lim_{x\to0}\frac{2-\frac{2}{\sigma\upsilon\nu^2 2x}}{-e^x+\frac{1}{1-x}}=\lim_{x\to0}\frac{\frac{2\sigma\upsilon\nu^2 2x-2}{\sigma\upsilon\nu^2 2x}}{\frac{-e^x(1-x)+1}{1-x}}=$$

$$=\lim_{x\to0}\frac{2\left(\sigma\upsilon\nu^2 2x-1\right)(1-x)}{\sigma\upsilon\nu^2 2x\left[e^x(x-1)+1\right]}=\lim_{x\to0}\frac{2(1-x)}{\sigma\upsilon\nu^2 2x}\cdot\lim_{x\to0}\frac{\sigma\upsilon\nu^2 2x-1}{e^x(x-1)+1}=$$

$$=\frac{2}{1}\cdot\lim_{x\to0}\frac{\sigma\upsilon\nu^2 2x-1}{e^x(x-1)+1}\overset{\frac{0}{0}}{\underset{DLH}{=}}2\cdot\lim_{x\to0}\frac{2\sigma\upsilon\nu 2x(-\eta\mu 2x)\cdot2}{e^x(x-1)+e^x}=2\cdot\lim_{x\to0}\frac{-4\sigma\upsilon\nu 2x\eta\mu 2x}{e^x x-\cancel{e^x}+\cancel{e^x}}=$$

$$=2\cdot\lim_{x\to0}\frac{-4\sigma\upsilon\nu 2x}{e^x}\cdot\lim_{x\to0}\frac{\eta\mu 2x}{x}=2\cdot\frac{-4}{1}\cdot\lim_{x\to0}\frac{\eta\mu 2x}{x}\overset{\frac{0}{0}}{\underset{DLH}{=}}-8\lim_{x\to0}\frac{2\sigma\upsilon\nu 2x}{1}==\boxed{-16}$$

18.4 11)

$$\boxed{\lim_{x\to+\infty}\frac{e^x}{x^{10}}}\overset{\frac{+\infty}{+\infty}}{\underset{DLH}{=}}\lim_{x\to+\infty}\frac{e^x}{10x^9}\overset{\frac{+\infty}{+\infty}}{\underset{DLH}{=}}\lim_{x\to+\infty}\frac{e^x}{10\cdot9\cdot x^8}\overset{\frac{+\infty}{+\infty}}{\underset{DLH}{=}}\cdots\overset{\frac{+\infty}{+\infty}}{\underset{DLH}{=}}\lim_{x\to+\infty}\frac{e^x}{10\cdot9\cdot8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}=$$

$$=\frac{+\infty}{10\cdot9\cdot8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}=\boxed{+\infty}$$

18.4 12)

$$\boxed{\lim_{x\to+\infty}\frac{\ln\left(4+e^{4x}\right)}{\ln\left(2+e^{2x}\right)}}\overset{\frac{+\infty}{+\infty}}{\underset{DLH}{=}}\lim_{x\to+\infty}\frac{\frac{\overset{2}{\cancel{4}}\,e^{\overset{2x}{\cancel{4x}}}}{4+e^{4x}}}{\frac{\cancel{2}\,\cancel{e^{2x}}}{2+e^{2x}}}=\lim_{x\to+\infty}\frac{2e^{2x}\left(2+e^{2x}\right)}{4+e^{4x}}=\lim_{x\to+\infty}\frac{4e^{2x}+2e^{4x}}{4+e^{4x}}=$$

$$=\lim_{x\to+\infty}\frac{\cancel{e^{4x}}\left(\frac{4}{e^{2x}}+2\right)}{\cancel{e^{4x}}\left(\frac{4}{e^{4x}}+1\right)}=\frac{\frac{4}{+\infty}+2}{\frac{4}{+\infty}+1}=\frac{0+2}{0+1}=\boxed{2}$$

18.4 13)

$$\boxed{\lim_{x\to0}\frac{\ln(\sigma\upsilon\nu 2x)}{\ln(\sigma\upsilon\nu 5x)}}=\lim_{x\to0}\frac{\frac{-\eta\mu 2x\cdot2}{\sigma\upsilon\nu 2x}}{\frac{-\eta\mu 5x\cdot5}{\sigma\upsilon\nu 5x}}=\lim_{x\to0}\frac{-2\eta\mu 2x\sigma\upsilon\nu 5x}{-5\eta\mu 5x\sigma\upsilon\nu 2x}=\lim_{x\to0}\frac{-2\sigma\upsilon\nu 5x}{-5\sigma\upsilon\nu 2x}\cdot\lim_{x\to0}\frac{\eta\mu 2x}{\eta\mu 5x}=$$

$$=\frac{-2\cdot1}{-5\cdot1}\cdot\lim_{x\to0}\frac{\eta\mu 2x}{\eta\mu 5x}\overset{\frac{0}{0}}{\underset{DLH}{=}}\frac{2}{5}\cdot\lim_{x\to0}\frac{2\sigma\upsilon\nu 2x}{5\sigma\upsilon\nu 5x}=\frac{2}{5}\cdot\frac{2\sigma\upsilon\nu 0}{5\sigma\upsilon\nu 0}=\boxed{\frac{4}{25}}$$

18.4 14)

$$\boxed{\lim_{x\to\pi^+}\frac{\sigma\upsilon\nu x\ln(x-\pi)}{\ln\left(e^x-e^\pi\right)}}=\lim_{x\to\pi^+}\sigma\upsilon\nu x\cdot\lim_{x\to\pi^+}\frac{\ln(x-\pi)}{\ln\left(e^x-e^\pi\right)}=\sigma\upsilon\nu\pi\cdot\lim_{x\to\pi^+}\frac{\ln(x-\pi)}{\ln\left(e^x-e^\pi\right)}\overset{\frac{+\infty}{+\infty}}{\underset{DLH}{=}}$$

$$=-1\cdot\lim_{x\to\pi^+}\frac{\frac{1}{x-\pi}}{\frac{e^x}{e^x-e^\pi}}=-1\cdot\lim_{x\to\pi^+}\frac{e^x-e^\pi}{e^x(x-\pi)}=-1\cdot\lim_{x\to\pi^+}\frac{1}{e^x}\cdot\lim_{x\to\pi^+}\frac{e^x-e^\pi}{x-\pi}=$$

$$=-\frac{1}{e^\pi}\cdot\lim_{x\to\pi^+}\frac{e^x-e^\pi}{x-\pi}\overset{\frac{0}{0}}{\underset{DLH}{=}}-\frac{1}{e^\pi}\cdot\lim_{x\to\pi^+}\frac{e^x}{1}=-\frac{1}{\cancel{e^\pi}}\cdot\cancel{e^\pi}=\boxed{-1}$$

$$\boxed{\lim_{x\to+\infty}\frac{\ln\left(1+e^x\right)}{5\sqrt{1+x^2}}} \underset{DLH}{\overset{\frac{+\infty}{+\infty}}{=}} \lim_{x\to+\infty}\frac{\frac{e^x}{1+e^x}}{5\cdot\frac{\cancel{2}x}{\cancel{2}\sqrt{1+x^2}}} = \lim_{x\to+\infty}\frac{e^x\sqrt{1+x^2}}{\left(1+e^x\right)5x} =$$

$$= \lim_{x\to+\infty}\frac{\sqrt{1+x^2}}{5x}\cdot\lim_{x\to+\infty}\frac{e^x}{1+e^x} = \lim_{x\to+\infty}\frac{\cancel{x}\sqrt{\frac{1}{x^2}+1}}{5\cancel{x}}\cdot\lim_{x\to+\infty}\frac{e^x}{1+e^x} = \frac{1}{5}\lim_{x\to+\infty}\frac{e^x}{1+e^x} \underset{DLH}{\overset{\frac{+\infty}{+\infty}}{=}}$$

$$= \frac{1}{5}\lim_{x\to+\infty}\frac{\cancel{e^x}}{\cancel{e^x}} = \boxed{\frac{1}{5}}$$