

17.3 1)

Είναι

$$y = z^2 - 1 \xrightarrow{z=3x+5} y(x) = (3x+5)^2 - 1 \Rightarrow y(x) = 9x^2 + 30x + 25 - 1 \Rightarrow$$

$$\Rightarrow y(x) = 9x^2 + 30x + 24 \Rightarrow \boxed{y'(x) = 18x + 30}$$

Οπότε

α) $y'(3) = 18 \cdot 3 + 30 \Rightarrow \boxed{y'(3) = 84}$

β) $y'(1) = 18 \cdot 1 + 30 \Rightarrow \boxed{y'(1) = 48}$

γ) Είναι $z = 3x + 5 \xrightarrow{z=2} 2 = 3x + 5 \Rightarrow x = -1$

Οπότε

$$y'(-1) = 18 \cdot (-1) + 30 \Rightarrow \boxed{y'(-1) = 12}$$

δ) Είναι $z = 3x + 5 \xrightarrow{z=5} 5 = 3x + 5 \Rightarrow x = 0$

Οπότε

$$y'(0) = 18 \cdot 0 + 30 \Rightarrow \boxed{y'(0) = 30}$$

17.3 2)

Είναι

$$y = z^2 + 2z \xrightarrow{x=z-1 \Rightarrow z=x+1} y(x) = (x+1)^2 + 2(x+1) \Rightarrow y(x) = x^2 + 2x + 1 + 2x + 2 \Rightarrow$$

$$\Rightarrow y(x) = x^2 + 4x + 3 \Rightarrow \boxed{y'(x) = 2x + 4}$$

Οπότε

α) $y'(4) = 2 \cdot 4 + 4 \Rightarrow \boxed{y'(4) = 12}$

β) $y'(0) = 2 \cdot 0 + 4 \Rightarrow \boxed{y'(0) = 4}$

γ) Είναι $x = z - 1 \xrightarrow{z=2} x = 2 - 1 \Rightarrow x = 1$

Οπότε

$$y'(1) = 2 \cdot 1 + 4 \Rightarrow \boxed{y'(1) = 6}$$

δ) Είναι $x = z - 1 \xrightarrow{z=-1} x = -1 - 1 \Rightarrow x = -2$

Οπότε

$$y'(-2) = 2 \cdot (-2) + 4 \Rightarrow \boxed{y'(-2) = 0}$$

17.3 3)

Είναι

$$y = 2z^2 + z - 2 \xrightarrow{z=x+2} y(x) = 2(x+2)^2 + x + \cancel{z} - \cancel{z} \Rightarrow y(x) = 2x^2 + 8x + 8 + x \Rightarrow$$

$$\Rightarrow y(x) = 2x^2 + 9x + 8 \Rightarrow \boxed{y'(x) = 4x + 9}$$

Οπότε

α) Είναι $z = x + 2 \xRightarrow{z=2} 2 = x + 2 \Rightarrow x = 0$

Οπότε

$$y'(0) = 4 \cdot 0 + 9 \Rightarrow \boxed{y'(0) = 9}$$

β) Είναι $z = x + 2 \xRightarrow{z=1} 1 = x + 2 \Rightarrow x = -1$

Οπότε

$$y'(-1) = 4 \cdot (-1) + 9 \Rightarrow \boxed{y'(-1) = 5}$$

γ) Είναι $z = x + 2 \xRightarrow{z=-1} -2 = x + 2 \Rightarrow x = -4$

Οπότε

$$y'(-4) = 4 \cdot (-4) + 9 \Rightarrow \boxed{y'(-4) = -7}$$

δ) Είναι $z = x + 2 \xRightarrow{z=-3} -3 = x + 2 \Rightarrow x = -5$

Οπότε

$$y'(-5) = 4 \cdot (-5) + 9 \Rightarrow \boxed{y'(-5) = -11}$$

17.3 4)

Είναι

$$y = z^3 - 2z^2 + 4z - 1 \xRightarrow{z=2x-7} y(x) = (2x-7)^3 - 2(2x-7)^2 + 4(2x-7) - 1 \Rightarrow$$

$$\Rightarrow \boxed{y'(x) = 6(2x-7)^2 - 8(2x-7) + 8}$$

Οπότε

α) Είναι $z = 2x - 7 \xRightarrow{z=-1} -1 = 2x - 7 \Rightarrow 2x = 6 \Rightarrow x = 3$

Οπότε

$$y'(3) = 6(2 \cdot 3 - 7)^2 - 8(2 \cdot 3 - 7) + 8 \Rightarrow y'(3) = 6 \cdot 1^2 - 8 \cdot (-1) + 8 \Rightarrow \boxed{y'(3) = 22}$$

β) Είναι $z = 2x - 7 \xRightarrow{z=3} 3 = 2x - 7 \Rightarrow 2x = 10 \Rightarrow x = 5$

Οπότε

$$y'(5) = 6(2 \cdot 5 - 7)^2 - 8(2 \cdot 5 - 7) + 8 \Rightarrow y'(5) = 6 \cdot 3^2 - 8 \cdot 3 + 8 \Rightarrow \boxed{y'(5) = 38}$$

γ) Είναι $z = 2x - 7 \xRightarrow{z=-7} -7 = 2x - 7 \Rightarrow 2x = 0 \Rightarrow x = 0$

Οπότε

$$y'(0) = 6(2 \cdot 0 - 7)^2 - 8(2 \cdot 0 - 7) + 8 \Rightarrow y'(0) = 6 \cdot (-7)^2 - 8 \cdot (-7) + 8 \Rightarrow \boxed{y'(0) = 358}$$

δ) Είναι $z = 2x - 7 \xRightarrow{z=1} 1 = 2x - 7 \Rightarrow 2x = 8 \Rightarrow x = 4$

Οπότε

$$y'(4) = 6(2 \cdot 4 - 7)^2 - 8(2 \cdot 4 - 7) + 8 \Rightarrow y'(4) = 6 \cdot 1^2 - 8 \cdot 1 + 8 \Rightarrow \boxed{y'(4) = 6}$$