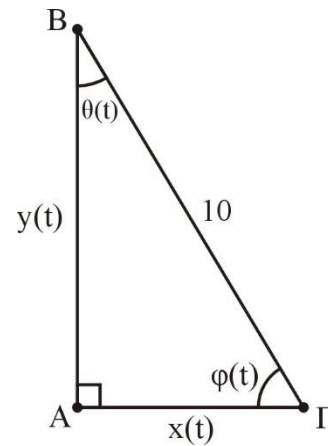


ΔΕΔΟΜΕΝΑ	ΖΗΤΟΥΜΕΝΑ
$x^2(t) + y^2(t) = 100$	α) $x(t_0) = ?$
$x'(t) = 12$	β) $y'(t_0) = ?$
$y(t_0) = 8$	γ) $\varphi'(t_0) = ?$
	δ) $\theta'(t_0) = ?$
	ε) $E'(t_0) = ?$



α) $x^2(t) + y^2(t) = 100 \xrightarrow{t=t_0} x^2(t_0) + y^2(t_0) = 100 \xrightarrow{y(t_0)=8} x^2(t_0) + 64 = 100 \Rightarrow$
 $\Rightarrow x^2(t_0) = 36 \Rightarrow x(t_0) = \pm\sqrt{36} \xrightarrow{x(t_0)>0} \boxed{x(t_0) = 6}$

β) $x^2(t) + y^2(t) = 100 \xrightarrow{\text{παραγωγίζουμε}} 2x(t)x'(t) + 2y(t)y'(t) = 0 \xrightarrow{t=t_0}$
 $\xrightarrow{\substack{x(t_0)=6 \\ x'(t_0)=12 \\ y(t_0)=8}} 2x(t_0)x'(t_0) + 2y(t_0)y'(t_0) = 0 \Rightarrow 2 \cdot 6 \cdot 12 + 2 \cdot 8 \cdot y'(t_0) = 0 \Rightarrow$
 $\Rightarrow 16y'(t_0) = -144 \Rightarrow \boxed{y'(t_0) = -9}$

γ) Είναι $\eta_{\mu\omega} = \frac{\text{απέναντι κάθετη}}{\text{υποτείνουσα}}$. Οπότε
 $\eta_{\mu\omega}(t) = \frac{y(t)}{10} \xrightarrow{\text{παραγωγίζουμε}} \text{συν}\varphi(t) \cdot \varphi'(t) = \frac{y'(t)}{10} \xrightarrow{t=t_0}$
 $\Rightarrow \text{συν}\varphi(t_0) \cdot \varphi'(t_0) = \frac{y'(t_0)}{10} \xrightarrow{\text{συν}\varphi(t_0) = \frac{x(t_0)}{10}} \frac{x(t_0)}{10} \cdot \varphi'(t_0) = \frac{y'(t_0)}{10} \Rightarrow$
 $\Rightarrow \varphi'(t_0) = \frac{y'(t_0)}{x(t_0)} \xrightarrow{\substack{y'(t_0)=-9 \\ x(t_0)=6}} \varphi'(t_0) = \frac{-9}{6} \Rightarrow \boxed{\varphi'(t_0) = -\frac{3}{2}}$

δ) Έχουμε ότι

$\varphi(t) + \theta(t) = \frac{\pi}{2} \xrightarrow{\text{παραγωγίζουμε}} \varphi'(t) + \theta'(t) = 0 \xrightarrow{t=t_0} \varphi'(t_0) + \theta'(t_0) = 0 \xrightarrow{\varphi'(t_0) = -\frac{3}{2}}$
 $-\frac{3}{2} + \theta'(t_0) = 0 \Rightarrow \boxed{\theta'(t_0) = \frac{3}{2}}$

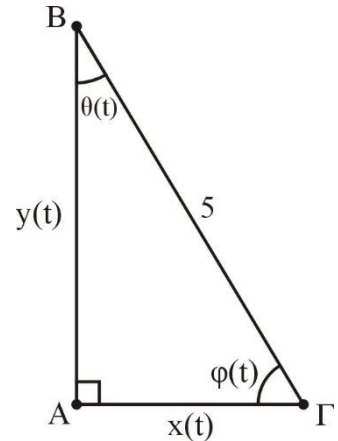
ε) Είναι

$E_{\text{τρίγωνου}} = \frac{\beta \cdot \upsilon}{2} \Rightarrow E(t) = \frac{x(t) \cdot y(t)}{2} \xrightarrow{\text{παραγωγίζουμε}}$
 $\Rightarrow E'(t) = \frac{x'(t) \cdot y(t) + x(t) \cdot y'(t)}{2} \xrightarrow{t=t_0} E'(t_0) = \frac{x'(t_0) \cdot y(t_0) + x(t_0) \cdot y'(t_0)}{2}$

$$\begin{aligned} x(t_0) &= 6 \\ x'(t_0) &= 12 \\ y(t_0) &= 8 \\ y'(t_0) &= -9 \\ \Rightarrow E'(t_0) &= \frac{12 \cdot 8 + 6 \cdot (-9)}{2} \Rightarrow \boxed{E'(t_0) = 21} \end{aligned}$$

17.14 2)

ΔΕΔΟΜΕΝΑ	ΖΗΤΟΥΜΕΝΑ
$x^2(t) + y^2(t) = 25$	α) $x(t_0) = ;$
$y'(t) = 8$	β) $y'(t_0) = ;$
$y(t_0) = 3$	γ) $\varphi'(t_0) = ;$
	δ) $\theta'(t_0) = ;$
	ε) $E'(t_0) = ;$



α) $x^2(t) + y^2(t) = 25 \xrightarrow{t=t_0} x^2(t_0) + y^2(t_0) = 25 \xrightarrow{y(t_0)=3} x^2(t_0) + 9 = 25 \Rightarrow$
 $\Rightarrow x^2(t_0) = 16 \Rightarrow x(t_0) = \pm\sqrt{16} \xrightarrow{x(t_0)>0} \boxed{x(t_0) = 4}$

β) $x^2(t) + y^2(t) = 25 \xrightarrow{\text{παραγωγίζουμε}} 2x(t)x'(t) + 2y(t)y'(t) = 0 \xrightarrow{t=t_0}$
 $\xrightarrow{\substack{x(t_0)=4 \\ y'(t_0)=8 \\ y(t_0)=3}} 2 \cdot 4 \cdot x'(t_0) + 2 \cdot 3 \cdot 8 = 0 \Rightarrow$
 $\Rightarrow 8x'(t_0) = -48 \Rightarrow \boxed{x'(t_0) = -6}$

γ) Είναι $\eta\mu\omega = \frac{\text{απέναντι κάθετη}}{\text{υποτείνουσα}}$. Οπότε

$$\begin{aligned} \eta\mu\varphi(t) &= \frac{y(t)}{5} \xrightarrow{\text{παραγωγίζουμε}} \sigma\upsilon\nu\varphi(t) \cdot \varphi'(t) = \frac{y'(t)}{5} \xrightarrow{t=t_0} \\ \Rightarrow \sigma\upsilon\nu\varphi(t_0) \cdot \varphi'(t_0) &= \frac{y'(t_0)}{5} \xrightarrow{\sigma\upsilon\nu\varphi(t_0) = \frac{x(t_0)}{5}} \frac{x(t_0)}{5} \cdot \varphi'(t_0) = \frac{y'(t_0)}{5} \Rightarrow \\ \Rightarrow \varphi'(t_0) &= \frac{y'(t_0)}{x(t_0)} \xrightarrow{\substack{y'(t_0)=8 \\ x(t_0)=4}} \varphi'(t_0) = \frac{8}{4} \Rightarrow \boxed{\varphi'(t_0) = 2} \end{aligned}$$

δ) Έχουμε ότι

$$\begin{aligned} \varphi(t) + \theta(t) &= \frac{\pi}{2} \xrightarrow{\text{παραγωγίζουμε}} \varphi'(t) + \theta'(t) = 0 \xrightarrow{t=t_0} \varphi'(t_0) + \theta'(t_0) = 0 \xrightarrow{\varphi'(t_0)=2} \\ 2 + \theta'(t_0) &= 0 \Rightarrow \boxed{\theta'(t_0) = -2} \end{aligned}$$

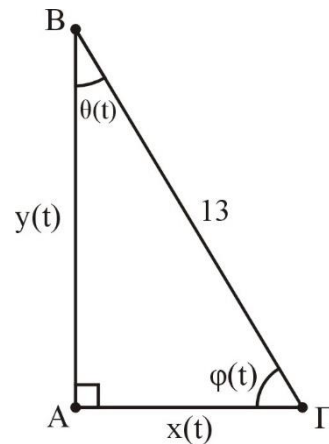
ε) Είναι

$$\begin{aligned} E_{\text{τρίγωνου}} &= \frac{\beta \cdot \upsilon}{2} \Rightarrow E(t) = \frac{x(t) \cdot y(t)}{2} \xrightarrow{\text{παραγωγίζουμε}} \\ \Rightarrow E'(t) &= \frac{x'(t) \cdot y(t) + x(t) \cdot y'(t)}{2} \xrightarrow{t=t_0} E'(t_0) = \frac{x'(t_0) \cdot y(t_0) + x(t_0) \cdot y'(t_0)}{2} \end{aligned}$$

$$\begin{aligned} & \begin{matrix} x(t_0)=4 \\ x'(t_0)=-6 \\ y(t_0)=3 \\ y'(t_0)=8 \end{matrix} \\ \Rightarrow E'(t_0) &= \frac{(-6) \cdot 3 + 4 \cdot 8}{2} \Rightarrow E'(t_0) = \frac{-18 + 32}{2} \Rightarrow \boxed{E'(t_0) = 7} \end{aligned}$$

17.14 3)

ΔΕΔΟΜΕΝΑ	ΖΗΤΟΥΜΕΝΑ
$x^2(t) + y^2(t) = 169$	α) $y(t_0) = ?$
$y'(t) = -10$	β) $x'(t_0) = ?$
$x(t_0) = 5$	γ) $\varphi'(t_0) = ?$
	δ) $\theta'(t_0) = ?$
	ε) $E'(t_0) = ?$



α) $x^2(t) + y^2(t) = 169 \xrightarrow{t=t_0} x^2(t_0) + y^2(t_0) = 169 \xrightarrow{x(t_0)=5} 25 + y^2(t_0) = 169 \Rightarrow$
 $\Rightarrow y^2(t_0) = 144 \Rightarrow y(t_0) = \pm \sqrt{144} \xrightarrow{y(t_0)>0} \boxed{y(t_0) = 12}$

β) $x^2(t) + y^2(t) = 169 \xrightarrow{\text{παραγωγίζουμε}} 2x(t)x'(t) + 2y(t)y'(t) = 0 \xrightarrow{t=t_0}$
 $\Rightarrow 2x(t_0)x'(t_0) + 2y(t_0)y'(t_0) = 0 \xrightarrow{\begin{matrix} x(t_0)=5 \\ y(t_0)=12 \\ y'(t_0)=-10 \end{matrix}} 2 \cdot 5 \cdot x'(t_0) + 2 \cdot 12 \cdot (-10) = 0 \Rightarrow$
 $\Rightarrow 10x'(t_0) = 240 \Rightarrow \boxed{x'(t_0) = 24}$

γ) Είναι $\eta\mu\omega = \frac{\text{απέναντι κάθετη}}{\text{υποτείνουσα}}$. Οπότε

$$\begin{aligned} \eta\mu\varphi(t) &= \frac{y(t)}{13} \xrightarrow{\text{παραγωγίζουμε}} \sigma\upsilon\nu\varphi(t) \cdot \varphi'(t) = \frac{y'(t)}{13} \xrightarrow{t=t_0} \\ \Rightarrow \sigma\upsilon\nu\varphi(t_0) \cdot \varphi'(t_0) &= \frac{y'(t_0)}{13} \xrightarrow{\sigma\upsilon\nu\varphi(t_0)=\frac{x(t_0)}{13}} \frac{x(t_0)}{13} \cdot \varphi'(t_0) = \frac{y'(t_0)}{13} \Rightarrow \\ \Rightarrow \varphi'(t_0) &= \frac{y'(t_0)}{x(t_0)} \xrightarrow{\begin{matrix} y'(t_0)=-10 \\ x(t_0)=5 \end{matrix}} \varphi'(t_0) = \frac{-10}{5} \Rightarrow \boxed{\varphi'(t_0) = -2} \end{aligned}$$

δ) Έχουμε ότι

$$\begin{aligned} \varphi(t) + \theta(t) &= \frac{\pi}{2} \xrightarrow{\text{παραγωγίζουμε}} \varphi'(t) + \theta'(t) = 0 \xrightarrow{t=t_0} \varphi'(t_0) + \theta'(t_0) = 0 \xrightarrow{\varphi'(t_0)=-2} \\ -2 + \theta'(t_0) &= 0 \Rightarrow \boxed{\theta'(t_0) = 2} \end{aligned}$$

ε) Είναι

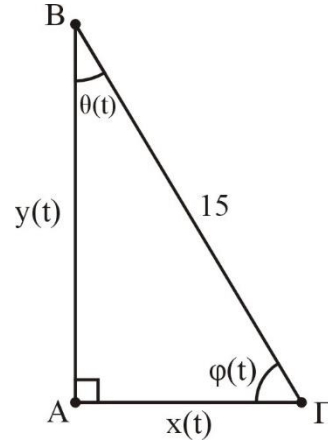
$$E_{\text{τρίγωνου}} = \frac{\beta \cdot \upsilon}{2} \Rightarrow E(t) = \frac{x(t) \cdot y(t)}{2} \xrightarrow{\text{παραγωγίζουμε}}$$

$$\Rightarrow E'(t) = \frac{x'(t) \cdot y(t) + x(t) \cdot y'(t)}{2} \xrightarrow{t=t_0} E'(t_0) = \frac{x'(t_0) \cdot y(t_0) + x(t_0) \cdot y'(t_0)}{2}$$

$$\begin{array}{l} x(t_0)=5 \\ x'(t_0)=24 \\ y(t_0)=12 \\ y'(t_0)=-10 \end{array} \Rightarrow E'(t_0) = \frac{24 \cdot 12 + 5 \cdot (-10)}{2} \Rightarrow E'(t_0) = \frac{288 - 50}{2} \Rightarrow \boxed{E'(t_0) = 119}$$

17.14 4)

ΔΕΔΟΜΕΝΑ	ΖΗΤΟΥΜΕΝΑ
$x^2(t) + y^2(t) = 225$	α) $y(t_0) = ;$
$x'(t) = -3$	β) $y'(t_0) = ;$
$x(t_0) = 12$	γ) $\varphi'(t_0) = ;$
	δ) $\theta'(t_0) = ;$
	ε) $E'(t_0) = ;$



α) $x^2(t) + y^2(t) = 225 \xrightarrow{t=t_0} x^2(t_0) + y^2(t_0) = 225 \xrightarrow{x(t_0)=12} 144 + y^2(t_0) = 225 \Rightarrow$
 $\Rightarrow y^2(t_0) = 225 - 144 \Rightarrow y^2(t_0) = 81 \Rightarrow y(t_0) = \pm \sqrt{81} \xrightarrow{y(t_0) > 0} \boxed{y(t_0) = 9}$

β) $x^2(t) + y^2(t) = 225 \xrightarrow{\text{παραγωγίζουμε}} 2x(t)x'(t) + 2y(t)y'(t) = 0 \xrightarrow{t=t_0}$
 $\xrightarrow{\begin{array}{l} x(t_0)=12 \\ x'(t_0)=-3 \\ y(t_0)=9 \end{array}} 2 \cdot 12 \cdot (-3) + 2 \cdot 9 \cdot y'(t_0) = 0 \Rightarrow$
 $\Rightarrow 18y'(t_0) = 72 \Rightarrow \boxed{y'(t_0) = 4}$

γ) Είναι $\eta\mu\omega = \frac{\text{απέναντι κάθετη}}{\text{υποτείνουσα}}$. Οπότε

$$\eta\mu\varphi(t) = \frac{y(t)}{15} \xrightarrow{\text{παραγωγίζουμε}} \sigma\upsilon\nu\varphi(t) \cdot \varphi'(t) = \frac{y'(t)}{15} \xrightarrow{t=t_0}$$

$$\Rightarrow \sigma\upsilon\nu\varphi(t_0) \cdot \varphi'(t_0) = \frac{y'(t_0)}{15} \xrightarrow{\sigma\upsilon\nu\varphi(t_0) = \frac{x(t_0)}{15}} \frac{x(t_0)}{15} \cdot \varphi'(t_0) = \frac{y'(t_0)}{15} \Rightarrow$$

$$\Rightarrow \varphi'(t_0) = \frac{y'(t_0)}{x(t_0)} \xrightarrow{\begin{array}{l} y'(t_0)=4 \\ x(t_0)=12 \end{array}} \varphi'(t_0) = \frac{4}{12} \Rightarrow \boxed{\varphi'(t_0) = \frac{1}{3}}$$

δ) Έχουμε ότι

$$\varphi(t) + \theta(t) = \frac{\pi}{2} \xrightarrow{\text{παραγωγίζουμε}} \varphi'(t) + \theta'(t) = 0 \xrightarrow{t=t_0} \varphi'(t_0) + \theta'(t_0) = 0 \xrightarrow{\varphi'(t_0) = \frac{1}{3}}$$

$$\frac{1}{3} + \theta'(t_0) = 0 \Rightarrow \boxed{\theta'(t_0) = -\frac{1}{3}}$$

ε) Είναι

$$E_{\text{τριγώνου}} = \frac{\beta \cdot \upsilon}{2} \Rightarrow E(t) = \frac{x(t) \cdot y(t)}{2} \xrightarrow{\text{παραγωγίζουμε}}$$

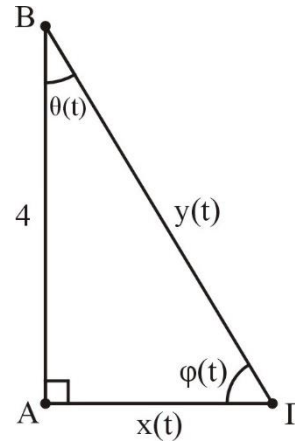
$$\Rightarrow E'(t) = \frac{x'(t) \cdot y(t) + x(t) \cdot y'(t)}{2} \xrightarrow{t=t_0} E'(t_0) = \frac{x'(t_0) \cdot y(t_0) + x(t_0) \cdot y'(t_0)}{2}$$

$$\begin{aligned} x(t_0) &= 12 \\ x'(t_0) &= -3 \\ y(t_0) &= 9 \\ y'(t_0) &= 4 \end{aligned}$$

$$\Rightarrow E'(t_0) = \frac{-3 \cdot 9 + 12 \cdot 4}{2} \Rightarrow E'(t_0) = \frac{-27 + 48}{2} \Rightarrow \boxed{E'(t_0) = \frac{21}{2}}$$

17.14 5)

ΔΕΔΟΜΕΝΑ	ΖΗΤΟΥΜΕΝΑ
$y^2(t) - x^2(t) = 16$	α) $y(t_0) = ?$
$x'(t) = 20$	β) $y'(t_0) = ?$
$x(t_0) = 3$	γ) $\varphi'(t_0) = ?$
	δ) $\theta'(t_0) = ?$
	ε) $E'(t_0) = ?$



α) $y^2(t) - x^2(t) = 16 \xrightarrow{t=t_0} y^2(t_0) - x^2(t_0) = 16 \xrightarrow{x(t_0)=3} y^2(t_0) - 3^2 = 16 \Rightarrow$

$$\Rightarrow y^2(t_0) = 16 + 9 \Rightarrow y^2(t_0) = 25 \Rightarrow y(t_0) = \pm \sqrt{25} \xrightarrow{y(t_0) > 0} \boxed{y(t_0) = 5}$$

β) $y^2(t) - x^2(t) = 16 \xrightarrow{\text{παραγωγίζουμε}} 2y(t)y'(t) - 2x(t)x'(t) = 0 \xrightarrow{t=t_0}$

$$\xrightarrow{\begin{matrix} x(t_0)=3 \\ x'(t_0)=20 \\ y(t_0)=5 \end{matrix}} 2y(t_0)y'(t_0) - 2x(t_0)x'(t_0) = 0 \Rightarrow 2 \cdot 5 \cdot y'(t_0) - 2 \cdot 3 \cdot 20 = 0 \Rightarrow$$

$$\Rightarrow 10y'(t_0) = 120 \Rightarrow \boxed{y'(t_0) = 12}$$

γ) Είναι $\eta\mu\omega = \frac{\text{απέναντι κάθετη}}{\text{υποτείνουσα}}$. Οπότε

$$\eta\mu\varphi(t) = \frac{4}{y(t)} \xrightarrow{\text{παραγωγίζουμε}} \text{συν}\varphi(t) \cdot \varphi'(t) = -\frac{4y'(t)}{y^2(t)} \xrightarrow{t=t_0}$$

$$\Rightarrow \text{συν}\varphi(t_0) \cdot \varphi'(t_0) = -\frac{4y'(t_0)}{y^2(t_0)} \xrightarrow{\text{συν}\varphi(t_0) = \frac{x(t_0)}{y(t_0)}} \frac{x(t_0)}{y(t_0)} \cdot \varphi'(t_0) = -\frac{4y'(t_0)}{y^2(t_0)} \Rightarrow$$

$$\Rightarrow \varphi'(t_0) = -\frac{4y'(t_0)}{x(t_0)y(t_0)} \xrightarrow{\begin{matrix} x(t_0)=3 \\ y(t_0)=5 \\ y'(t_0)=12 \end{matrix}} \varphi'(t_0) = -\frac{4 \cdot 12}{3 \cdot 5} \Rightarrow \boxed{\varphi'(t_0) = -\frac{16}{5}}$$

δ) Έχουμε ότι

$$\varphi(t) + \theta(t) = \frac{\pi}{2} \xrightarrow{\text{παραγωγίζουμε}} \varphi'(t) + \theta'(t) = 0 \xrightarrow{t=t_0} \varphi'(t_0) + \theta'(t_0) = 0 \Rightarrow \xrightarrow{\varphi'(t_0) = -\frac{16}{5}}$$

$$-\frac{16}{5} + \theta'(t_0) = 0 \Rightarrow \boxed{\theta'(t_0) = \frac{16}{5}}$$

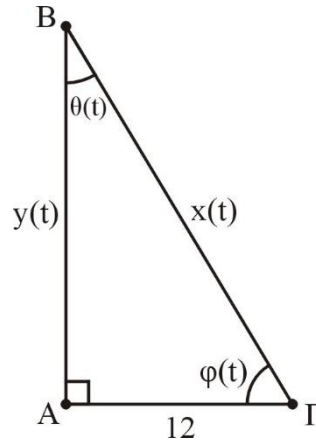
ε) Είναι

$$E_{\text{τριγώνου}} = \frac{\beta \cdot v}{2} \Rightarrow E(t) = \frac{\cancel{4} \cdot x(t)}{\cancel{2}} \xrightarrow{\text{παραγωγίζουμε}} \Rightarrow E'(t) = 2x'(t) \Rightarrow$$

$$\xrightarrow{t=t_0} \Rightarrow E'(t_0) = 2x'(t_0) \xrightarrow{x'(t_0)=20} \boxed{E'(t_0) = 40}$$

17.14 6)

ΔΕΔΟΜΕΝΑ	ΖΗΤΟΥΜΕΝΑ
$x^2(t) - y^2(t) = 144$	α) $x(t_0) = ?$
$x'(t) = 15$	β) $y'(t_0) = ?$
$y(t_0) = 5$	γ) $\varphi'(t_0) = ?$
	δ) $\theta'(t_0) = ?$
	ε) $E'(t_0) = ?$



α) $x^2(t) - y^2(t) = 144 \xrightarrow{t=t_0} x^2(t_0) - y^2(t_0) = 144 \xrightarrow{y(t_0)=5} x^2(t_0) - 5^2 = 144 \Rightarrow$
 $\Rightarrow x^2(t_0) = 144 + 25 \Rightarrow x^2(t_0) = 169 \Rightarrow x(t_0) = \pm\sqrt{169} \xrightarrow{x(t_0)>0} \boxed{x(t_0) = 13}$

β) $x^2(t) - y^2(t) = 144 \xrightarrow{\text{παραγωγίζουμε}} 2x(t)x'(t) - 2y(t)y'(t) = 0 \xrightarrow{t=t_0}$
 $\Rightarrow 2x(t_0)x'(t_0) - 2y(t_0)y'(t_0) = 0 \xrightarrow{\substack{x(t_0)=13 \\ x'(t_0)=15 \\ y(t_0)=5}} 2 \cdot 13 \cdot 15 - 2 \cdot 5 \cdot y'(t_0) = 0 \Rightarrow$
 $\Rightarrow 10y'(t_0) = 390 \Rightarrow \boxed{y'(t_0) = 39}$

γ) Είναι $\text{συν}\omega = \frac{\text{προσκεείμενη κάθετη}}{\text{υποτείνουσα}}$. Οπότε

$$\text{συν}\varphi(t) = \frac{12}{x(t)} \xrightarrow{\text{παραγωγίζουμε}} \cancel{\eta\mu\varphi(t)} \cdot \varphi'(t) = \cancel{\frac{12x'(t)}{x^2(t)}} \xrightarrow{t=t_0}$$

$$\Rightarrow \eta\mu\varphi(t_0) \cdot \varphi'(t_0) = \frac{12x'(t_0)}{x^2(t_0)} \xrightarrow{\eta\mu\varphi(t_0) = \frac{y(t_0)}{x(t_0)}} \frac{y(t_0)}{\cancel{x(t_0)}} \cdot \varphi'(t_0) = \frac{12x'(t_0)}{\cancel{x^2(t_0)}} \Rightarrow$$

$$\Rightarrow \varphi'(t_0) = \frac{12x'(t_0)}{x(t_0) \cdot y(t_0)} \xrightarrow{\substack{x'(t_0)=15 \\ x(t_0)=13 \\ y(t_0)=5}} \varphi'(t_0) = \frac{12 \cdot \cancel{15}}{13 \cdot \cancel{5}} \Rightarrow \boxed{\varphi'(t_0) = \frac{36}{13}}$$

δ) Έχουμε ότι

$$\varphi(t) + \theta(t) = \frac{\pi}{2} \xrightarrow{\text{παραγωγίζουμε}} \varphi'(t) + \theta'(t) = 0 \xrightarrow{t=t_0} \varphi'(t_0) + \theta'(t_0) = 0 \Rightarrow$$

$$\frac{36}{13} + \theta'(t_0) = 0 \Rightarrow \boxed{\theta'(t_0) = -\frac{36}{13}}$$

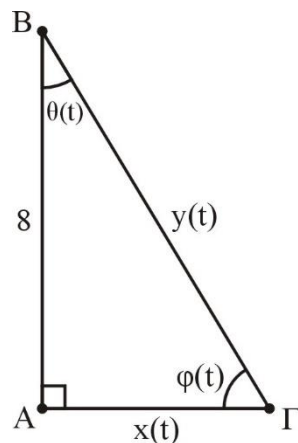
ε) Είναι

$$E_{\text{τριγώνου}} = \frac{\beta \cdot v}{2} \Rightarrow E(t) = \frac{\cancel{12} \cdot y(t)}{\cancel{2}} \xrightarrow{\text{παραγωγίζουμε}} \Rightarrow E'(t) = 6y'(t) \Rightarrow$$

$$\xrightarrow{t=t_0} \Rightarrow E'(t_0) = 6y'(t_0) \xrightarrow{y'(t_0)=39} \boxed{E'(t_0) = 234}$$

17.14 7)

ΔΕΔΟΜΕΝΑ	ΖΗΤΟΥΜΕΝΑ
$y^2(t) - x^2(t) = 64$	α) $y(t_0) = ?$
$y'(t) = -9$	β) $x'(t_0) = ?$
$x(t_0) = 6$	γ) $\varphi'(t_0) = ?$
	δ) $\theta'(t_0) = ?$
	ε) $E'(t_0) = ?$



$$\alpha) \quad y^2(t) - x^2(t) = 64 \xrightarrow{t=t_0} y^2(t_0) - x^2(t_0) = 64 \xrightarrow{x(t_0)=6} y^2(t_0) - 36 = 64 \Rightarrow$$

$$\Rightarrow y^2(t_0) = 64 + 36 \Rightarrow y^2(t_0) = 100 \Rightarrow y(t_0) = \pm \sqrt{100} \xrightarrow{y(t_0) > 0} \boxed{y(t_0) = 10}$$

$$\beta) \quad y^2(t) - x^2(t) = 64 \xrightarrow{\text{παραγωγίζουμε}} 2y(t)y'(t) - 2x(t)x'(t) = 0 \xrightarrow{t=t_0}$$

$$\xrightarrow{\substack{x(t_0)=6 \\ y(t_0)=10 \\ y'(t_0)=-9}} \Rightarrow 2y(t_0)y'(t_0) - 2x(t_0)x'(t_0) = 0 \Rightarrow 2 \cdot 10 \cdot (-9) - 2 \cdot 6 \cdot x'(t_0) = 0 \Rightarrow$$

$$\Rightarrow -180 - 12 \cdot x'(t_0) = 0 \Rightarrow 12x'(t_0) = -180 \Rightarrow \boxed{x'(t_0) = -15}$$

$$\gamma) \quad \text{Είναι } \eta_{\mu\omega} = \frac{\text{απέναντι κάθετη}}{\text{υποτείνουσα}}. \text{ Οπότε}$$

$$\eta_{\mu\omega}(t) = \frac{8}{y(t)} \xrightarrow{\text{παραγωγίζουμε}} \text{συν}\varphi(t) \cdot \varphi'(t) = -\frac{8y'(t)}{y^2(t)} \xrightarrow{t=t_0}$$

$$\Rightarrow \text{συν}\varphi(t_0) \cdot \varphi'(t_0) = -\frac{8y'(t_0)}{y^2(t_0)} \xrightarrow{\text{συν}\varphi(t_0) = \frac{x(t_0)}{y(t_0)}} \frac{x(t_0)}{y(t_0)} \cdot \varphi'(t_0) = -\frac{8y'(t_0)}{y^2(t_0)} \Rightarrow$$

$$\Rightarrow \varphi'(t_0) = -\frac{8y'(t_0)}{x(t_0)y(t_0)} \xrightarrow{\substack{x(t_0)=6 \\ y(t_0)=10 \\ y'(t_0)=-9}} \varphi'(t_0) = -\frac{8 \cdot (-9)}{6 \cdot 10} \Rightarrow \varphi'(t_0) = \frac{72}{60} \Rightarrow \boxed{\varphi'(t_0) = \frac{6}{5}}$$

$$\delta) \quad \text{Έχουμε ότι}$$

$$\varphi(t) + \theta(t) = \frac{\pi}{2} \xrightarrow{\text{παραγωγίζουμε}} \varphi'(t) + \theta'(t) = 0 \xrightarrow{t=t_0} \varphi'(t_0) + \theta'(t_0) = 0 \xrightarrow{\varphi'(t_0) = \frac{6}{5}} \Rightarrow$$

$$\frac{6}{5} + \theta'(t_0) = 0 \Rightarrow \boxed{\theta'(t_0) = -\frac{6}{5}}$$

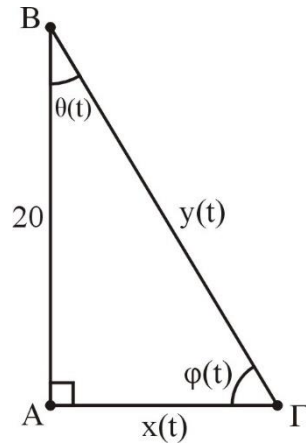
$$\epsilon) \quad \text{Είναι}$$

$$E_{\text{τριγώνου}} = \frac{\beta \cdot v}{2} \Rightarrow E(t) = \frac{\cancel{4} \cdot x(t)}{\cancel{2}} \xrightarrow{\text{παραγωγίζουμε}} \Rightarrow E'(t) = 4x'(t) \Rightarrow$$

$$\begin{aligned} & \stackrel{t=t_0}{\Rightarrow} E'(t_0) = 4x'(t_0) \stackrel{x'(t_0)=-15}{\Rightarrow} \boxed{E'(t_0) = -60} \end{aligned}$$

17.14 8)

ΔΕΔΟΜΕΝΑ	ΖΗΤΟΥΜΕΝΑ
$y^2(t) - x^2(t) = 400$	α) $x(t_0) = ?$
$x'(t) = -5$	β) $y'(t_0) = ?$
$y(t_0) = 25$	γ) $\varphi'(t_0) = ?$
	δ) $\theta'(t_0) = ?$
	ε) $E'(t_0) = ?$



$$\begin{aligned} \alpha) \quad y^2(t) - x^2(t) &= 400 \stackrel{t=t_0}{\Rightarrow} y^2(t_0) - x^2(t_0) = 400 \stackrel{y(t_0)=25}{\Rightarrow} 25^2 - x^2(t_0) = 400 \Rightarrow \\ &\Rightarrow 625 - x^2(t_0) = 400 \Rightarrow x^2(t_0) = 225 \Rightarrow x(t_0) = \pm \sqrt{225} \stackrel{x(t_0)>0}{\Rightarrow} \boxed{x(t_0) = 15} \end{aligned}$$

$$\begin{aligned} \beta) \quad y^2(t) - x^2(t) &= 400 \stackrel{\text{παραγωγίζουμε}}{\Rightarrow} 2y(t)y'(t) - 2x(t)x'(t) = 0 \stackrel{t=t_0}{\Rightarrow} \\ &\stackrel{\substack{x(t_0)=15 \\ x'(t_0)=-5 \\ y(t_0)=25}}{\Rightarrow} 2 \cdot 25 \cdot y'(t_0) - 2 \cdot 15 \cdot (-5) = 0 \Rightarrow \\ &\Rightarrow 50 \cdot y'(t_0) + 150 = 0 \Rightarrow 50y'(t_0) = -150 \Rightarrow \boxed{y'(t_0) = -3} \end{aligned}$$

γ) Είναι $\eta_{\mu\omega} = \frac{\text{απέναντι κάθετη}}{\text{υποτείνουσα}}$. Οπότε

$$\begin{aligned} \eta_{\mu\omega}(t) &= \frac{20}{y(t)} \stackrel{\text{παραγωγίζουμε}}{\Rightarrow} \text{συν}\varphi(t) \cdot \varphi'(t) = -\frac{20y'(t)}{y^2(t)} \stackrel{t=t_0}{\Rightarrow} \\ &\Rightarrow \text{συν}\varphi(t_0) \cdot \varphi'(t_0) = -\frac{20y'(t_0)}{y^2(t_0)} \stackrel{\text{συν}\varphi(t_0)=\frac{x(t_0)}{y(t_0)}}{\Rightarrow} \frac{x(t_0)}{y(t_0)} \cdot \varphi'(t_0) = -\frac{20y'(t_0)}{y^2(t_0)} \Rightarrow \\ &\Rightarrow \varphi'(t_0) = -\frac{20y'(t_0)}{x(t_0)y(t_0)} \stackrel{\substack{x(t_0)=15 \\ y(t_0)=25 \\ y'(t_0)=-3}}{\Rightarrow} \varphi'(t_0) = -\frac{20 \cdot (-3)}{15 \cdot 25} \Rightarrow \boxed{\varphi'(t_0) = \frac{4}{25}} \end{aligned}$$

δ) Έχουμε ότι

$$\begin{aligned} \varphi(t) + \theta(t) &= \frac{\pi}{2} \stackrel{\text{παραγωγίζουμε}}{\Rightarrow} \varphi'(t) + \theta'(t) = 0 \stackrel{t=t_0}{\Rightarrow} \varphi'(t_0) + \theta'(t_0) = 0 \stackrel{\varphi'(t_0)=\frac{4}{25}}{\Rightarrow} \\ &\frac{4}{25} + \theta'(t_0) = 0 \Rightarrow \boxed{\theta'(t_0) = -\frac{4}{25}} \end{aligned}$$

ε) Είναι

$$E_{\text{τριγώνου}} = \frac{\beta \cdot \nu}{2} \Rightarrow E(t) = \frac{20 \cdot x(t)}{2} \stackrel{\text{παραγωγίζουμε}}{\Rightarrow} E'(t) = 10x'(t) \Rightarrow$$

$$\overset{t=t_0}{\Rightarrow} E'(t_0) = 10x'(t_0) \overset{x'(t_0)=-5}{\Rightarrow} \boxed{E'(t_0) = -50}$$