

Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Α

17.11 1)

Τα δεδομένα και τα ζητούμενα της άσκησης είναι

Δεδομένα	Ζητούμενα
$x^2(t) + y^2(t) = 25$	α) $y(t_0) = ;$
$x(t_0) = -3$	β) $y'(t_0) = ;$
$x'(t_0) = 8$	
$y(t_0) < 0$	

Οπότε

$$\alpha) \quad x^2(t) + y^2(t) = 25 \xrightarrow{\text{θέτουμε } t=t_0} x^2(t_0) + y^2(t_0) = 25 \xrightarrow{x(t_0)=-3} (-3)^2 + y^2(t_0) = 25 \Rightarrow$$

$$\Rightarrow 9 + y^2(t_0) = 25 \Rightarrow y^2(t_0) = 16 \xrightarrow{y(t_0)<0} y(t_0) = -4$$

$$\beta) \quad x^2(t) + y^2(t) = 25 \xrightarrow{\text{παραγωγίζουμε}} 2x(t)x'(t) + 2y(t)y'(t) = 0 \xrightarrow{t=t_0}$$

$$\Rightarrow 2x(t_0)x'(t_0) + 2y(t_0)y'(t_0) = 0 \xrightarrow{x(t_0)=-3, y(t_0)=-4, x'(t_0)=8} 2(-3) \cdot 8 + 2(-4)y'(t_0) = 0 \Rightarrow$$

$$-24 - 4y'(t_0) = 0 \Rightarrow y'(t_0) = -6$$

17.11 2)

ΔΕΔΟΜΕΝΑ	ΖΗΤΟΥΜΕΝΑ
$y(t) = x^2(t)$	$y'(t_0) = ;$
$x(t_0) = 2$	
$x'(t_0) = 1$	

Έχουμε

$$y(t) = x^2(t) \xrightarrow{\text{παραγωγίζουμε}} y'(t) = 2x(t)x'(t) \xrightarrow{t=t_0} y'(t_0) = 2x(t_0)x'(t_0) \Rightarrow$$

$$\xrightarrow{x(t_0)=2, x'(t_0)=1} y'(t_0) = 2 \cdot 2 \cdot 1 \Rightarrow \boxed{y'(t_0) = 4}$$

17.11 3)

ΔΕΔΟΜΕΝΑ	ΖΗΤΟΥΜΕΝΑ
$y(t) = x^3(t) - 2x^2(t) + 1$	$y'(t_0) = ;$
$x(t_0) = 1$	
$x'(t_0) = 2$	

Έχουμε

$$y(t) = x^3(t) - 2x^2(t) + 1 \xrightarrow{\text{παραγωγίζουμε}} y'(t) = 3x^2(t)x'(t) - 4x(t)x'(t) \xrightarrow{t=t_0}$$

$$\Rightarrow y'(t_0) = 3x^2(t_0)x'(t_0) - 4x(t_0)x'(t_0) \xrightarrow{x(t_0)=1, x'(t_0)=2} y'(t_0) = 3 \cdot 1^2 \cdot 2 - 4 \cdot 1 \cdot 2 \Rightarrow$$

$$\Rightarrow \boxed{y'(t_0) = -2}$$

17.11 4)

ΔΕΔΟΜΕΝΑ	ΖΗΤΟΥΜΕΝΑ
$y(t) = x(t) + e^{x(t)-4}$	$y'(t_0) = ;$
$x(t_0) = 4$	
$x'(t_0) = -1$	

Έχουμε

$$y(t) = x(t) + e^{x(t)-4} \xRightarrow{\text{παραγωγίζουμε}} y'(t) = x'(t) + e^{x(t)-4} x'(t) \xRightarrow{t=t_0}$$

$$\Rightarrow y'(t_0) = x'(t_0) + e^{x(t_0)-4} x'(t_0) \xRightarrow{x(t_0)=4, x'(t_0)=-1} y'(t_0) = -1 + e^{4-4}(-1) \Rightarrow$$

$$\Rightarrow \boxed{y'(t_0) = -2}$$

17.11 5)

Τα δεδομένα και τα ζητούμενα της άσκησης είναι

Δεδομένα	Ζητούμενα
$x^2(t) + y^2(t) = 25$	α) $y(t_0) = ;$
$x(t_0) = 4$	β) $x'(t_0) = ;$
$y'(t_0) = 20$	
$y(t_0) > 0$	

Οπότε

$$\alpha) \quad x^2(t) + y^2(t) = 25 \xRightarrow{\text{θέτουμε } t=t_0} x^2(t_0) + y^2(t_0) = 25 \xRightarrow{x(t_0)=4} 4^2 + y^2(t_0) = 25 \Rightarrow$$

$$\Rightarrow 16 + y^2(t_0) = 25 \Rightarrow y^2(t_0) = 9 \xRightarrow{y(t_0)>0} y(t_0) = 3$$

$$\beta) \quad x^2(t) + y^2(t) = 25 \xRightarrow{\text{παραγωγίζουμε}} 2x(t)x'(t) + 2y(t)y'(t) = 0 \xRightarrow{t=t_0}$$

$$\Rightarrow 2x(t_0)x'(t_0) + 2y(t_0)y'(t_0) = 0 \xRightarrow{x(t_0)=4, y(t_0)=3, y'(t_0)=20} 2 \cdot 4 \cdot x'(t) + 2 \cdot 3 \cdot 20 = 0 \Rightarrow$$

$$8x'(t) + 120 = 0 \Rightarrow x'(t_0) = -15$$

17.11 6)

Τα δεδομένα και τα ζητούμενα της άσκησης είναι

Δεδομένα	Ζητούμενα
$x^2(t) + y^2(t) = 100$	α) $y(t_0) = ;$
$y(t_0) = -6$	β) $x'(t_0) = ;$
$x'(t_0) = -3$	
$x(t_0) < 0$	

Οπότε

$$\alpha) \quad x^2(t) + y^2(t) = 100 \xRightarrow{\text{θέτουμε } t=t_0} x^2(t_0) + y^2(t_0) = 100 \xRightarrow{y(t_0)=-6}$$

$$\Rightarrow x^2(t_0) + (-6)^2 = 100 \Rightarrow x^2(t_0) + 36 = 100 \Rightarrow x^2(t_0) = 64 \stackrel{x(t_0) < 0}{\Rightarrow} x(t_0) = -8$$

$$\textbf{β)} \quad x^2(t) + y^2(t) = 100 \quad \stackrel{\text{παραγωγίζουμε}}{\Rightarrow} \quad 2x(t)x'(t) + 2y(t)y'(t) = 0 \stackrel{t=t_0}{\Rightarrow}$$

$$\Rightarrow 2x(t_0)x'(t_0) + 2y(t_0)y'(t_0) = 0 \quad \stackrel{\substack{x(t_0)=-8, \quad y(t_0)=-6 \\ x'(t_0)=-3}}{\Rightarrow}$$

$$\Rightarrow 2 \cdot (-8) \cdot (-3) + 2 \cdot (-6) \cdot y'(t_0) = 0 \Rightarrow 48 - 12y'(t_0) = 0 \Rightarrow y'(t_0) = 4$$