

# Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Β

## 31.9 1)

$$\begin{aligned} \text{α)} \quad \int_1^8 \frac{1}{\sqrt{3x+1}} dx &= \int_4^{25} \frac{1}{3} \frac{1}{\sqrt{y}} dy = \frac{1}{3} \left[ 2\sqrt{y} \right]_4^{25} = \\ &= \frac{1}{3} (2\sqrt{25} - 2\sqrt{4}) = \frac{1}{3} (10 - 4) = 2 \end{aligned}$$

θέτουμε  $y=3x+1 \Rightarrow dy=3dx \Rightarrow dx=\frac{1}{3}dy$   
 για  $x=1 \Rightarrow y=3 \cdot 1 + 1 = 4$   
 για  $x=8 \Rightarrow y=3 \cdot 8 + 1 = 25$

$$\begin{aligned} \text{β)} \quad \int_1^{\sqrt{13}} x\sqrt{x^2+3} dx &= \int_4^{16} \frac{1}{2} \sqrt{y} dy = \frac{1}{2} \int_4^{16} y^{\frac{1}{2}} dy = \\ &= \frac{1}{2} \left[ \frac{y^{\frac{3}{2}}}{\frac{3}{2}} \right]_4^{16} = \frac{1}{2} \cdot \frac{2}{3} \left[ \sqrt{y^3} \right]_4^{16} = \frac{1}{3} \left[ \sqrt{16^3} - \sqrt{4^3} \right] = \frac{1}{3} (64 - 8) = \frac{56}{3} \end{aligned}$$

θέτουμε  $y=x^2+3 \Rightarrow dy=2x dx \Rightarrow x dx = \frac{1}{2} dy$   
 για  $x=1 \Rightarrow y=1^2+3=4$   
 για  $x=\sqrt{13} \Rightarrow y=\sqrt{13}^2+3=16$

## 31.9 2)

$$\int_0^3 \sqrt{x+1} dx = \int_1^4 \sqrt{y} dy = \left[ \frac{y^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^4 = \frac{1}{3} \left( 4^{\frac{3}{2}} - 1^{\frac{3}{2}} \right) = \frac{2}{3} (8 - 1) = \frac{14}{3}$$

θέτουμε  $y=x+1 \Rightarrow dy=dx \Rightarrow dx=dy$   
 για  $x=0 \Rightarrow y=1$   
 για  $x=3 \Rightarrow y=4$

## 31.9 3)

$$\begin{aligned} \int_1^2 \sqrt{3x-2} dx &= \int_1^4 \sqrt{y} \frac{1}{3} dy = \frac{1}{3} \int_1^4 y^{\frac{1}{2}} dy = \frac{1}{3} \left[ \frac{y^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^4 = \\ &= \frac{1}{3} \frac{1}{3} \left( 4^{\frac{3}{2}} - 1^{\frac{3}{2}} \right) = \frac{2}{9} (\sqrt{4^3} - 1) = \frac{2}{9} \cdot 7 = \frac{14}{9} \end{aligned}$$

θέτουμε  $y=3x-2 \Rightarrow dy=3dx \Rightarrow dx=\frac{1}{3}dy$   
 για  $x=1 \Rightarrow y=1$   
 για  $x=2 \Rightarrow y=4$

## 31.9 4)

$$\int_0^1 \frac{1}{\sqrt{2x+1}} dx = \int_1^3 \frac{1}{\sqrt{y}} \frac{1}{2} dy = \left[ \sqrt{y} \right]_1^3 = \sqrt{3} - 1$$

θέτουμε  $y=2x+1 \Rightarrow dy=2dx \Rightarrow dx=\frac{1}{2}dy$   
 για  $x=0 \Rightarrow y=1$   
 για  $x=1 \Rightarrow y=3$

## 31.9 5)

$$\int_0^{-\frac{1}{2}} \sqrt[3]{2x+1} dx = \int_1^0 y^{\frac{1}{3}} \frac{1}{2} dy = \frac{1}{2} \left[ \frac{y^{\frac{4}{3}}}{\frac{4}{3}} \right]_1^0 = \frac{1}{2} \left( \frac{0^{\frac{4}{3}}}{\frac{4}{3}} - \frac{1}{\frac{4}{3}} \right) = \frac{1}{2} \cdot \frac{-3}{4} = -\frac{3}{8}$$

θέτουμε  $y=2x+1 \Rightarrow dy=2dx \Rightarrow dx=\frac{1}{2}dy$   
 για  $x=0 \Rightarrow y=1$   
 για  $x=-\frac{1}{2} \Rightarrow y=0$

## 31.9 6)

$$\int_0^{\frac{7}{2}} \frac{1}{\sqrt[3]{1-2x}} dx = \int_1^{-8} \frac{1}{\sqrt[3]{y}} \left( -\frac{1}{2} \right) dy = -\frac{1}{2} \int_1^{-8} y^{-\frac{1}{3}} dy =$$

θέτουμε  $y=1-2x \Rightarrow dy=-2dx \Rightarrow dx=-\frac{1}{2}dy$   
 για  $x=0 \Rightarrow y=1$   
 για  $x=-\frac{7}{2} \Rightarrow y=-8$

$$-\frac{1}{2} \left[ \frac{y^{\frac{2}{3}}}{\frac{2}{3}} \right]_0^8 = -\frac{1}{2} \frac{1}{\frac{2}{3}} \left( \sqrt[3]{8^2} - 1^{\frac{2}{3}} \right) = -\frac{3}{4} (4 - 1) = -\frac{9}{4}$$

**31.9 7)**

$$\begin{array}{l} \text{θέτουμε } y=x^2+4 \Rightarrow dy=2x dx \Rightarrow x dx = \frac{1}{2} dy \\ \text{για } x=0 \Rightarrow y=4 \\ \text{για } x=2\sqrt{3} \Rightarrow y=16 \end{array}$$

$$\int_0^{2\sqrt{3}} x \sqrt{x^2+4} dx = \int_4^{16} \sqrt{y} \frac{1}{2} dy = \frac{1}{2} \int_4^{16} y^{\frac{1}{2}} dy =$$

$$\frac{1}{2} \left[ \frac{y^{\frac{3}{2}}}{\frac{3}{2}} \right]_4^{16} = \frac{1}{2} \frac{1}{\frac{3}{2}} \left( 16^{\frac{3}{2}} - 4^{\frac{3}{2}} \right) = \frac{1}{3} (64 - 8) = \frac{56}{3}$$

**31.9 8)**

$$\begin{array}{l} \text{θέτουμε } y=x^3+x^2+x+1 \Rightarrow dy=(3x^2+2x+1) dx \\ \text{για } x=0 \Rightarrow y=1 \\ \text{για } x=1 \Rightarrow y=4 \end{array}$$

$$\int_0^1 (3x^2+2x+1) \sqrt{x^3+x^2+x+1} dx = \int_1^4 \sqrt{y} dy = \left[ \frac{y^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^4 =$$

$$= \frac{1}{\frac{3}{2}} \left( 4^{\frac{3}{2}} - 1^{\frac{3}{2}} \right) = \frac{2}{3} (8 - 1) = \frac{14}{3}$$

**31.9 9)**

$$\begin{array}{l} \text{θέτουμε } y=x^3+2 \Rightarrow dy=3x^2 dx \Rightarrow x^2 dx = \frac{1}{3} dy \\ \text{για } x=-1 \Rightarrow y=1 \\ \text{για } x=\sqrt[3]{2} \Rightarrow y=4 \end{array}$$

$$\int_{-1}^{\sqrt[3]{2}} x^2 \sqrt{x^3+2} dx = \int_1^4 \sqrt{y} \frac{1}{3} dy = \frac{1}{3} \left[ \frac{y^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^4 =$$

$$= \frac{1}{3} \frac{1}{\frac{3}{2}} \left( 4^{\frac{3}{2}} - 1^{\frac{3}{2}} \right) = \frac{2}{9} (8 - 1) = \frac{14}{9}$$

**31.9 10)**

$$\begin{array}{l} \text{θέτουμε } y=4+x^2 \Rightarrow dy=2x dx \Rightarrow x dx = \frac{1}{2} dy \\ \text{για } x=0 \Rightarrow y=4 \\ \text{για } x=2\sqrt{3} \Rightarrow y=16 \end{array}$$

$$\int_0^{2\sqrt{3}} \frac{x}{\sqrt{4+x^2}} dx = \int_4^{16} \frac{1}{\sqrt{y}} \frac{1}{2} dy = \left[ \sqrt{y} \right]_4^{16} = \sqrt{16} - \sqrt{4} = 2$$

**31.9 11)**

$$\begin{array}{l} \text{θέτουμε } y=x^2+7x+1 \Rightarrow dy=(2x+7) dx \\ \text{για } x=0 \Rightarrow y=1 \\ \text{για } x=1 \Rightarrow y=9 \end{array}$$

$$\int_0^1 (2x+7) \sqrt{x^2+7x+1} dx = \int_1^9 \sqrt{y} dy = \left[ \frac{y^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^9 =$$

$$= \frac{1}{\frac{3}{2}} \left( 9^{\frac{3}{2}} - 1^{\frac{3}{2}} \right) = \frac{2}{3} (27 - 1) = \frac{52}{3}$$

**31.9 12)**

$$\int_{\sqrt{2}}^{\sqrt{7}} \frac{x}{\sqrt{x^2+2}} dx \quad \begin{array}{l} \text{θέτουμε } y=x^2+2 \Rightarrow dy=2x dx \Rightarrow x dx = \frac{1}{2} dy \\ \text{για } x=\sqrt{2} \Rightarrow y=4 \\ \text{για } x=\sqrt{7} \Rightarrow y=9 \end{array} = \int_4^9 \frac{1}{\sqrt{y}} \frac{1}{2} dy = \left[ \sqrt{y} \right]_4^9 = \sqrt{9} - \sqrt{4} = 3 - 2 = 1$$

**31.9 13)**

$$\int_{\sqrt[3]{2}}^{\sqrt[3]{7}} \frac{x^2}{\sqrt{x^3+2}} dx \quad \begin{array}{l} \text{θέτουμε } y=x^3+2 \Rightarrow dy=3x^2 dx \Rightarrow x^2 dx = \frac{1}{3} dy \\ \text{για } x=\sqrt[3]{2} \Rightarrow y=4 \\ \text{για } x=\sqrt[3]{7} \Rightarrow y=9 \end{array} = \int_4^9 \frac{1}{\sqrt{y}} \frac{1}{3} dy = \frac{1}{3} \left[ \frac{y^{\frac{1}{2}}}{\frac{1}{2}} \right]_4^9 = \frac{1}{3} \frac{1}{\frac{1}{2}} \left( 9^{\frac{1}{2}} - 4^{\frac{1}{2}} \right) =$$

$$= \frac{2}{3} (3 - 2) = \frac{2}{3}$$

**31.9 14)**

$$\int_0^{2\sqrt{2}} x \sqrt{x^2+1} dx \quad \begin{array}{l} \text{θέτουμε } y=x^2+1 \Rightarrow dy=2x dx \Rightarrow x dx = \frac{1}{2} dy \\ \text{για } x=2\sqrt{2} \Rightarrow y=9 \\ \text{για } x=0 \Rightarrow y=1 \end{array} = \int_1^9 \sqrt{y} \frac{1}{2} dy = \frac{1}{2} \left[ \frac{y^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^9 = \frac{1}{2} \frac{1}{\frac{3}{2}} \left( 9^{\frac{3}{2}} - 1^{\frac{3}{2}} \right) =$$

$$= \frac{1}{3} (27 - 1) = \frac{26}{3}$$

**31.9 15)**

$$\int_{\sqrt{2}}^{\sqrt{5}} x \sqrt{x^2-1} dx \quad \begin{array}{l} \text{θέτουμε } y=x^2-1 \Rightarrow dy=2x dx \Rightarrow x dx = \frac{1}{2} dy \\ \text{για } x=\sqrt{5} \Rightarrow y=4 \\ \text{για } x=\sqrt{2} \Rightarrow y=1 \end{array} = \int_1^4 \sqrt{y} \frac{1}{2} dy = \frac{1}{2} \left[ \frac{y^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^4 = \frac{1}{2} \frac{1}{\frac{3}{2}} \left( 4^{\frac{3}{2}} - 1^{\frac{3}{2}} \right) =$$

$$= \frac{1}{3} (8 - 1) = \frac{7}{3}$$

**31.9 16)**

$$\int_{-3}^1 \frac{2x+1}{\sqrt{x^2+x+2}} dx \quad \begin{array}{l} \text{θέτουμε } y=x^2+x+2 \Rightarrow dy=(2x+1) dx \\ \text{για } x=-3 \Rightarrow y=8 \\ \text{για } x=1 \Rightarrow y=4 \end{array} = \int_8^4 \frac{1}{\sqrt{y}} dy = \left[ \frac{y^{\frac{1}{2}}}{\frac{1}{2}} \right]_8^4 = \frac{1}{\frac{1}{2}} \left( 4^{\frac{1}{2}} - 8^{\frac{1}{2}} \right) =$$

$$= 2(2 - \sqrt{8}) = 2(2 - 2\sqrt{2}) = 4 - 4\sqrt{2}$$

**31.9 17)**

$$\int_0^{\sqrt{15}} \frac{2x}{\sqrt{x^2+1}} dx \quad \begin{array}{l} \text{θέτουμε } y=x^2+1 \Rightarrow dy=2x dx \\ \text{για } x=0 \Rightarrow y=1 \\ \text{για } x=\sqrt{15} \Rightarrow y=16 \end{array} = \int_1^{16} \frac{1}{\sqrt{y}} dy = \left[ \frac{y^{\frac{1}{2}}}{\frac{1}{2}} \right]_1^{16} = \frac{1}{\frac{1}{2}} \left( 16^{\frac{1}{2}} - 1^{\frac{1}{2}} \right) = 2(4 - 1) = 6$$

**31.9 18)**

$$\int_0^1 \frac{2x^3}{\sqrt{x^4+1}} dx \quad \begin{array}{l} \text{θέτουμε } y=x^4+1 \Rightarrow dy=4x^3 dx \Rightarrow \frac{1}{2} dy=2x^3 dx \\ \text{για } x=0 \Rightarrow y=1 \\ \text{για } x=1 \Rightarrow y=2 \end{array} = \int_1^2 \frac{1}{\sqrt{y}} \frac{1}{2} dy = \left[ \sqrt{y} \right]_1^2 = \sqrt{2} - 1$$

**31.9 19)**

$$\int_0^{\sqrt{7}} \frac{x}{\sqrt[3]{x^2+1}} dx$$

$$= \int_1^8 \frac{1}{\sqrt[3]{y}} \frac{1}{2} dy = \frac{1}{2} \int_1^8 y^{-\frac{1}{3}} dy = \frac{1}{2} \left[ \frac{y^{\frac{2}{3}}}{\frac{2}{3}} \right]_1^8 =$$

θέτουμε  $y=x^2+1 \Rightarrow dy=2x dx \Rightarrow x dx = \frac{1}{2} dy$   
 για  $x=0 \Rightarrow y=1$   
 για  $x=\sqrt{7} \Rightarrow y=8$

$$= \frac{1}{2} \frac{1}{2} \left( 8^{\frac{2}{3}} - 1^{\frac{2}{3}} \right) = \frac{3}{4} (4 - 1) = \frac{9}{4}$$