

Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Β

31.8 1)

$$\begin{aligned} \text{α)} \quad \int_0^1 (3x-1)^5 dx &= \int_{-1}^2 y^5 \frac{1}{3} dy = \frac{1}{3} \left[\frac{y^6}{6} \right]_{-1}^2 = \frac{1}{3} \left(\frac{2^6}{6} - \frac{1}{6} \right) = \frac{7}{2} \end{aligned}$$

θέτουμε $y=3x-1 \Rightarrow dy=3dx \Rightarrow dx=\frac{1}{3}dy$
 για $x=0 \Rightarrow y=3 \cdot 0 - 1 = -1$
 για $x=1 \Rightarrow y=3 \cdot 1 - 1 = 2$

$$\begin{aligned} \text{β)} \quad \int_{-1}^0 (x^2 + 3x + 4)^2 (2x + 3) dx &= \int_2^4 y^2 dy = \left[\frac{y^3}{3} \right]_2^4 = \end{aligned}$$

θέτουμε $y=x^2+3x+4 \Rightarrow dy=(2x+3)dx$
 για $x=-1 \Rightarrow y=(-1)^2+3(-1)+4=2$
 για $x=0 \Rightarrow y=0^2+3 \cdot 0 + 4 = 4$

$$\frac{64}{3} - \frac{8}{3} = \frac{56}{3}$$

$$\begin{aligned} \text{γ)} \quad \int_{-2}^1 \frac{3x}{(x^2+2)^2} dx &= 3 \int_{-2}^1 \frac{1}{(x^2+2)^2} x dx = 3 \int_6^3 \frac{1}{y^2} \frac{1}{2} dy = \\ &= \frac{3}{2} \int_6^3 \frac{1}{y^2} dy = \frac{3}{2} \left[-\frac{1}{y} \right]_6^3 = \frac{3}{2} \left[-\frac{1}{3} + \frac{1}{6} \right] = \frac{3}{2} \left(-\frac{1}{6} \right) = -\frac{1}{4} \end{aligned}$$

θέτουμε $y=x^2+2 \Rightarrow dy=2x dx \Rightarrow x dx = \frac{1}{2} dy$
 για $x=-2 \Rightarrow y=(-2)^2+2=6$
 για $x=1 \Rightarrow y=1^2+2=3$

31.8 2)

$$\begin{aligned} \int_{-1}^0 (2x+1)^5 dx &= \int_{-1}^1 y^5 \frac{1}{2} dy = \frac{1}{2} \left[\frac{y^6}{6} \right]_{-1}^1 = \frac{1}{2} \left(\frac{1}{6} - \frac{1}{6} \right) = 0 \end{aligned}$$

θέτουμε $y=2x+1 \Rightarrow dy=2dx \Rightarrow dx=\frac{1}{2}dy$
 για $x=-1 \Rightarrow y=-1$
 για $x=0 \Rightarrow y=1$

31.8 3)

$$\begin{aligned} \int_{\frac{1}{3}}^0 (3x-2)^5 dx &= \int_{-2}^{-1} y^5 \frac{1}{3} dy = \frac{1}{3} \left[\frac{y^6}{6} \right]_{-2}^{-1} = \frac{1}{3} \left(\frac{1}{6} - \frac{64}{6} \right) = \frac{7}{2} \end{aligned}$$

θέτουμε $y=3x-2 \Rightarrow dy=3dx \Rightarrow dx=\frac{1}{3}dy$
 για $x=\frac{1}{3} \Rightarrow y=-1$
 για $x=0 \Rightarrow y=-2$

31.8 4)

$$\begin{aligned} \int_{\frac{2}{5}}^{\frac{7}{5}} (5x-2)^4 dx &= \int_0^5 y^4 \frac{1}{5} dy = \frac{1}{5} \left[\frac{y^5}{5} \right]_0^5 = \frac{1}{5} \left(\frac{5^5}{5} - \frac{0}{5} \right) = 5^3 = 125 \end{aligned}$$

θέτουμε $y=5x-2 \Rightarrow dy=5dx \Rightarrow dx=\frac{1}{5}dy$
 για $x=\frac{2}{5} \Rightarrow y=0$
 για $x=\frac{7}{5} \Rightarrow y=5$

31.8 5)

$$\begin{aligned} \int_0^1 \frac{1}{(2x+1)^3} dx &= \int_1^3 \frac{1}{y^3} \frac{1}{2} dy = \frac{1}{2} \left[\frac{y^{-2}}{-2} \right]_1^3 = -\frac{1}{4} \left[\frac{1}{y^2} \right]_1^3 = \\ &= -\frac{1}{4} \left(\frac{1}{3^2} - \frac{1}{1^2} \right) = -\frac{1}{4} \left(\frac{1}{9} - 1 \right) = -\frac{1}{4} \cdot \frac{-8}{9} = \frac{2}{9} \end{aligned}$$

θέτουμε $y=2x+1 \Rightarrow dy=2dx \Rightarrow dx=\frac{1}{2}dy$
 για $x=0 \Rightarrow y=1$
 για $x=1 \Rightarrow y=3$

31.8 6)

$$\begin{aligned} \int_0^1 (x^3 + x - 1)^6 (3x^2 + 1) dx &= \int_{-1}^1 y^6 dy = \left[\frac{y^7}{7} \right]_{-1}^1 = \frac{1}{7} - \left(-\frac{1}{7} \right) = \frac{2}{7} \end{aligned}$$

θέτουμε $y=x^3+x-1 \Rightarrow dy=(3x^2+1)dx$
 για $x=0 \Rightarrow y=-1$
 για $x=1 \Rightarrow y=1$

31.8 7)

$$\int_1^0 \frac{2x-1}{(x^2-x+4)^{15}} dx \stackrel{\substack{\text{θέτουμε } y=x^2-x+4 \Rightarrow dy=(2x-1)dx \\ \text{για } x=1 \Rightarrow y=4 \\ \text{για } x=0 \Rightarrow y=4}}{=} \int_4^4 \frac{1}{y^{15}} dy = 0$$

31.8 8)

$$\int_0^1 \frac{x}{(x^2+1)^3} dx \stackrel{\substack{\text{θέτουμε } y=x^2+1 \Rightarrow dy=2x dx \Rightarrow x dx = \frac{1}{2} dy \\ \text{για } x=0 \Rightarrow y=1 \\ \text{για } x=1 \Rightarrow y=2}}{=} \int_1^2 \frac{1}{y^3} \frac{1}{2} dy = \frac{1}{2} \int_1^2 y^{-3} dy = \frac{1}{2} \left[\frac{y^{-2}}{-2} \right]_1^2 =$$

$$= -\frac{1}{4} (2^{-2} - 1^{-2}) = -\frac{1}{4} \left(\frac{1}{4} - 1 \right) = -\frac{1}{4} \cdot \frac{-3}{4} = \frac{3}{16}$$

31.8 9)

$$\int_0^1 \frac{2x+1}{x^2+x+1} dx \stackrel{\substack{\text{θέτουμε } y=x^2+x+1 \Rightarrow dy=(2x+1)dx \\ \text{για } x=0 \Rightarrow y=1 \\ \text{για } x=1 \Rightarrow y=3}}{=} \int_1^3 \frac{1}{y} dy = \left[\ln|x| \right]_1^3 = \ln 3 - \ln 1 = \ln 3$$

31.8 10)

$$\int_1^0 (x^2-2x)^{2000} (2x-2) dx \stackrel{\substack{\text{θέτουμε } y=x^2-2x \Rightarrow dy=(2x-2)dx \\ \text{για } x=1 \Rightarrow y=-1 \\ \text{για } x=0 \Rightarrow y=0}}{=} \int_{-1}^0 y^{2000} dy = \left[\frac{y^{2001}}{2001} \right]_{-1}^0 = 0 - \frac{-1}{2001} = \frac{1}{2001}$$

31.8 11)

$$\int_0^6 \frac{x-3}{\left(\frac{x^2}{2} - 3x + 5 \right)^{101}} dx \stackrel{\substack{\text{θέτουμε } y = \frac{x^2}{2} - 3x + 5 \Rightarrow dy = (x-3)dx \Rightarrow dx = \frac{1}{2} dy \\ \text{για } x=0 \Rightarrow y=5 \\ \text{για } x=6 \Rightarrow y=5}}{=} \int_5^5 \frac{1}{y^{101}} dy = 0$$