Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Β

31.7 1)

α) $\int_{-1}^{1}\frac{4x}{x^2+3}dx=2\int_{-1}^{1}\frac{2x}{x^2+3}dx\overset{x^2+3>0}{=}2\left[\ln\left(x^2+3\right)\right]_{-1}^{1}=2\left[\ln\left(1^2+3\right)-\ln\left(\left(-1\right)^2+3\right)\right]=$

$=2\left[\ln 4-\ln 4\right]=0$

β) $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\frac{\sigma\upsilon\nu x}{\sqrt{\eta\mu x}}dx=2\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\frac{\sigma\upsilon\nu x}{2\sqrt{\eta\mu x}}dx=2\left[\sqrt{\eta\mu x}\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}=2\left(\sqrt{\eta\mu\frac{\pi}{2}}-\sqrt{\eta\mu\frac{\pi}{6}}\right)=$

$=2\left(\sqrt{1}-\sqrt{\frac{1}{2}}\right)=2\left(1-\frac{1}{\sqrt{2}}\right)=2-\frac{2}{\sqrt{2}}=2-\frac{\cancel{2}\sqrt{2}}{\cancel{2}}=2-\sqrt{2}$

γ) $\int_{1}^{4}\frac{x^2+1}{\sqrt{x}}dx=\int_{1}^{4}\left(\frac{x^2}{\sqrt{x}}+\frac{1}{\sqrt{x}}\right)dx=\int_{1}^{4}\left(\frac{x^2}{x^{\frac{1}{2}}}+\frac{1}{x^{\frac{1}{2}}}\right)dx=\int_{1}^{4}\left(x^{\frac{3}{2}}+x^{-\frac{1}{2}}\right)dx=$

$=\int_{1}^{4}\left(x^{\frac{3}{2}}+x^{-\frac{1}{2}}\right)dx=\left[\frac{x^{\frac{5}{2}}}{\frac{5}{2}}+\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]_{1}^{4}=\left[\frac{2\sqrt{x}^5}{5}+2\sqrt{x}\right]_{1}^{4}=$

$=\frac{2\sqrt{4}^5}{5}+2\sqrt{4}-\left(\frac{2\sqrt{1}^5}{5}+2\sqrt{1}\right)=\frac{2\cdot 32}{5}+4-\frac{2}{5}-2=\frac{72}{5}$

31.7 2)

$\int_{0}^{2}\left(2xe^{x^2}\right)dx=\left[e^{x^2}\right]_{0}^{2}=e^4-e^0=e^4-1$

31.7 3)

$\int_{0}^{\frac{\pi}{2}}\left(\eta\mu x+x\sigma\upsilon\nu x\right)dx=\left[x\eta\mu x\right]_{0}^{\frac{\pi}{2}}=\frac{\pi}{2}\eta\mu\frac{\pi}{2}-0\eta\mu 0=\frac{\pi}{2}$

31.7 4)

$\int_{0}^{\ln 7}\frac{e^x}{e^x+2}dx\overset{e^x+2>0}{=}\left[\ln\left(e^x+2\right)\right]_{0}^{\ln 7}=\ln\left(e^{\ln 7}+2\right)-\ln\left(e^0+2\right)=\ln 9-\ln 3=\ln\frac{9}{3}=\ln 3$

31.7 5)

$\int_{\frac{1}{2}}^{1}\frac{1-\ln x}{x^2}dx=\left[\frac{\ln x}{x}\right]_{\frac{1}{2}}^{1}=\frac{\ln 1}{1}-\frac{\ln\frac{1}{2}}{\frac{1}{2}}=0-2\ln 2^{-1}=2\ln 2$

31.7 6)

$\int_{1}^{2}\frac{4x+2}{x^2+x+2}dx\overset{x^2+x+2>0}{=}\left[\ln\left(x^2+x+2\right)\right]_{1}^{2}=\ln\left(2^2+2+2\right)-\ln\left(1^2+1+2\right)=\ln 8-\ln 4=$

$=\ln\frac{8}{4}=\ln 2$

31.7 7)

$\int_{0}^{2\sqrt{2}}\frac{2x}{\sqrt{x^2+1}}dx=2\int_{0}^{2\sqrt{2}}\frac{2x}{2\sqrt{x^2+1}}dx=2\left[\sqrt{x^2+1}\right]_{0}^{2\sqrt{2}}=2\left(\sqrt{\left(2\sqrt{2}\right)^2+1}-\sqrt{0^2+1}\right)=$

$$=2\left(\sqrt{8+1}-1\right)=2(3-1)=4$$

31.7 8)

$$\int_1^{\frac{1}{2}}\frac{e^{2x}}{1-e^{2x}}dx=-\frac{1}{2}\int_1^{\frac{1}{2}}\frac{-2e^{2x}}{1-e^{2x}}dx=-\frac{1}{2}\left[\ln\left|1-e^{2x}\right|\right]_1^{\frac{1}{2}}=-\frac{1}{2}\left[\ln\left|1-e^{2\frac{1}{2}}\right|-\ln\left|1-e^{2\cdot 1}\right|\right]=$$

$$=-\frac{1}{2}\left[\ln|1-e|-\ln\left|1-e^2\right|\right]\overset{\substack{1-e<0\\1-e^2<0}}{=}-\frac{1}{2}\left[\ln(e-1)-\ln\left(e^2-1\right)\right]=-\frac{1}{2}\ln\frac{e-1}{e^2-1}=$$

$$=-\frac{1}{2}\ln\frac{1}{e+1}=\frac{1}{2}\ln(e+1)$$

31.7 9)

$$\int_0^3(2x+1)^2dx=\int_0^3\left(4x^2+4x+1\right)dx=\left[\frac{4x^3}{3}+\frac{4x^2}{2}+x\right]_0^3=\frac{4\cdot 3^{\cancel{3}^2}}{\cancel{3}}+\frac{\cancel{4}^2\cdot 3^2}{\cancel{2}}+3-0=$$

$$=36+18+3=57$$

31.7 10)

$$\int_2^4\frac{x^2-1}{x}dx=\int_2^4\left(\frac{x^{\cancel{2}}}{\cancel{x}}-\frac{1}{x}\right)dx=\int_2^4\left(x-\frac{1}{x}\right)dx=\left[\frac{x^2}{2}-\ln|x|\right]_2^4=$$

$$=\frac{4^2}{2}-\ln 4-\left(\frac{2^2}{2}-\ln 2\right)=8-\ln 4-2+\ln 2=6-2\ln 2+\ln 2=6-\ln 2$$

31.7 11)

$$\int_{-2}^{-6}\frac{x^2+x+1}{x}dx=\int_{-2}^{-6}\frac{x^{\cancel{2}}}{\cancel{x}}+\frac{x}{x}+\frac{1}{x}dx=\int_{-2}^{-6}\left(x+1+\frac{1}{x}\right)dx=\left[\frac{x^2}{2}+x+\ln|x|\right]_{-2}^{-6}=$$

$$=\frac{(-6)^2}{2}-6+\ln|-6|-\left(\frac{(-2)^2}{2}-2+\ln|-2|\right)=18-6+\ln 6-\cancel{2}+\cancel{2}-\ln 2=12+\ln 3$$

31.7 12)

$$\int_1^4\frac{x+1}{\sqrt{x}}dx=\int_1^4\left(\frac{x}{\sqrt{x}}+\frac{1}{\sqrt{x}}\right)dx=\int_1^4\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)dx=\int_1^4\sqrt{x}dx+\int_1^4\frac{1}{\sqrt{x}}dx=$$

$$=\int_1^4 x^{\frac{1}{2}}dx+2\int_1^4\frac{1}{2\sqrt{x}}dx=\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^4+2\left[\sqrt{x}\right]_1^4=\frac{2\cdot 4^{\frac{3}{2}}}{3}-\frac{2\cdot 1^{\frac{3}{2}}}{3}+2\left(\sqrt{4}-\sqrt{1}\right)=$$

$$=\frac{2\cdot\sqrt{4}^3}{3}-\frac{2\cdot 1}{3}+2(2-1)=\frac{16}{3}-\frac{2}{3}+2=\frac{20}{3}$$