31.60

Έστω $I=\int_0^{\frac{\pi}{4}}\frac{1}{\sigma\upsilon\nu^4 x}dx$. Τότε

$$I=\int_0^{\frac{\pi}{4}}\frac{1}{\sigma\upsilon\nu^4 x}dx=\int_0^{\frac{\pi}{4}}\frac{1}{\sigma\upsilon\nu^2 x}\cdot\frac{1}{\sigma\upsilon\nu^2 x}dx=\int_0^{\frac{\pi}{4}}\frac{1}{\sigma\upsilon\nu^2 x}(\varepsilon\varphi x)'\,dx=$$

$$=\left[\frac{1}{\sigma\upsilon\nu^2 x}\varepsilon\varphi x\right]_0^{\frac{\pi}{4}}-\int_0^{\frac{\pi}{4}}\left(\frac{1}{\sigma\upsilon\nu^2 x}\right)'\varepsilon\varphi x dx=$$

$$=\frac{1}{\sigma\upsilon\nu^2\frac{\pi}{4}}\varepsilon\varphi\frac{\pi}{4}-0-\int_0^{\frac{\pi}{4}}-\frac{-2\eta\mu x\,\cancel{\sigma\upsilon\nu x}}{\sigma\upsilon\nu^{\cancel{4}3}x}\varepsilon\varphi x dx=$$

$$=\frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2}-\int_0^{\frac{\pi}{4}}\frac{2\eta\mu x}{\sigma\upsilon\nu^3 x}\frac{\eta\mu x}{\sigma\upsilon\nu x}dx=\frac{1}{\frac{2}{4}}-2\int_0^{\frac{\pi}{4}}\frac{2\eta\mu^2 x}{\sigma\upsilon\nu^4 x}dx=$$

$$=2-2\left(\int_0^{\frac{\pi}{4}}\frac{1}{\sigma\upsilon\nu^4 x}dx-\int_0^{\frac{\pi}{4}}\frac{\cancel{\sigma\upsilon\nu^2 x}}{\sigma\upsilon\nu^{\cancel{4}2}x}dx\right)=2-2\left(I-\int_0^{\frac{\pi}{4}}\frac{1}{\sigma\upsilon\nu^2 x}dx\right)=$$

$$=2-2I+2[\varepsilon\varphi x]_0^{\frac{\pi}{4}}=2-2I+2\left(\varepsilon\varphi\frac{\pi}{4}-\varepsilon\varphi 0\right)=2-2I+2=4-2I$$

Επομένως

$$I=4-2I\Rightarrow 3I=4\Rightarrow I=\frac{4}{3}$$