Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Β

31.6 1)

α) $\int_0^1 (x^2-3x+1)dx = \left[\frac{x^3}{3}-\frac{3x^2}{2}+x\right]_0^1 = \frac{1^3}{3}-\frac{3\cdot 1^2}{2}+1-\left(\frac{0^3}{3}-\frac{3\cdot 0^2}{2}+0\right) = -\frac{1}{6}$

β) $\int_1^2 \frac{2}{x^3}dx = 2\int_1^2 x^{-3}dx = 2\left[\frac{x^{-2}}{-2}\right]_1^2 = 2\left(\frac{2^{-2}}{-2}-\frac{1^{-1}}{-2}\right) = -\frac{1}{4}+1 = \frac{3}{4}$

γ) $\int_0^9 (\sqrt{x}+1)dx = \int_0^9 \left(x^{\frac{1}{2}}+1\right)dx = \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+x\right]_0^9 = \left[\frac{2\sqrt{x}^3}{3}+x\right]_0^9 =$

$= \frac{2\sqrt{9}^3}{3}+9-\left(\frac{2\sqrt{0}^3}{3}+0\right) = 27$

δ) $\int_0^1 (e^x-4x)dx = \left[e^x-2x^2\right]_0^1 = e^1-2\cdot 1^2-\left(e^0-2\cdot 0^2\right) = e-2-1 = e-3$

31.6 2)

$\int_0^3 (x^2+1)dx = \left[\frac{x^3}{3}+x\right]_0^3 = \frac{3^3}{3}+3-\left(\frac{0^3}{3}+0\right) = 9+3 = 12$

31.6 3)

$\int_0^2 (2x^3-x+1)dx = \left[2\frac{x^4}{4}-\frac{x^2}{2}+x\right]_0^2 = 2\frac{2^4}{4}-\frac{2^2}{2}+2-\left(2\frac{0^4}{4}-\frac{0^2}{2}+0\right) =$

$2\cdot 4-2+2 = 8$

31.6 4)

$\int_1^2 \frac{4}{x^4}dx = 4\int_1^2 x^{-4}dx = 4\left[\frac{x^{-3}}{-3}\right]_1^2 = \frac{4}{-3}\left[\frac{1}{x^3}\right]_1^2 = -\frac{4}{3}\left(\frac{1}{2^3}-\frac{1}{1^3}\right) = -\frac{4}{3}\left(\frac{1}{8}-1\right) =$

$-\frac{4}{3}\left(\frac{1}{8}-1\right) = -\frac{4}{3}\frac{-7}{8} = \frac{7}{6}$

31.6 5)

$\int_{-1}^{-2} \frac{8}{x^3}dx = 8\int_{-1}^{-2} x^{-3}dx = 8\left[\frac{x^{-2}}{-2}\right]_{-1}^{-2} = -\frac{8}{2}\left[\frac{1}{x^2}\right]_{-1}^{-2} = -4\left[\frac{1}{(-2)^2}-\frac{1}{(-1)^2}\right] =$

$= -4\left(\frac{1}{4}-1\right) = -\cancel{4}\cdot\frac{-3}{\cancel{4}} = 3$

31.6 6)

$\int_0^1 \sqrt{x}dx = \int_0^1 x^{\frac{1}{2}}dx = \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1 = \frac{1^{\frac{3}{2}}}{\frac{3}{2}}-\frac{0^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2}{3}$

31.6 7)

$$\int_0^8 \sqrt[3]{x}\,dx = \int_0^8 x^{\frac{1}{3}}dx = \left[\frac{x^{\frac{4}{3}}}{\frac{4}{3}}\right]_0^8 = \frac{8^{\frac{4}{3}}}{\frac{4}{3}} - \frac{0^{\frac{4}{3}}}{\frac{4}{3}} = \frac{3\cdot 8^{\frac{4}{3}}}{4} = \frac{3\cdot \sqrt[3]{8}^4}{4} = \frac{3\cdot 2^4}{4} = 12$$

31.6 8)

$$\int_1^2 \frac{1}{6x}dx = \frac{1}{6}\int_1^2 \frac{1}{x}dx = \frac{1}{6}\left[\ln|x|\right]_1^2 = \frac{1}{6}\left(\ln|2| - \ln|1|\right) = \frac{\ln 2}{6}$$

31.6 9)

$$\int_{-2}^{-1} \frac{1}{2x}dx = \frac{1}{2}\int_{-2}^{-1} \frac{1}{x}dx = \frac{1}{2}\left[\ln|x|\right]_{-2}^{-1} = \frac{1}{2}\left[\ln|-1| - \ln|-2|\right] = -\frac{\ln 2}{2}$$

31.6 10)

$$\int_{-2}^{-4} \frac{4}{3x}dx = \frac{4}{3}\int_{-2}^{-4} \frac{1}{x}dx = \frac{4}{3}\left[\ln|x|\right]_{-2}^{-4} = \frac{4}{3}\left[\ln|x| - \ln|x|\right]_{-2}^{-4} = \frac{4}{3}\left(\ln|-4| - \ln|-2|\right) =$$

$$= \frac{4}{3}\left(\ln 4 - \ln 2\right) = \frac{4}{3}\ln\frac{4}{2} = \frac{4\ln 2}{3}$$

31.6 11)

$$\int_2^3 \frac{1}{2x-3}dx = \left[\frac{\ln|2x-3|}{2}\right]_2^3 = \frac{\ln|2\cdot 3-3|}{2} - \frac{\ln|2\cdot 2-3|}{2} = \frac{\ln 3}{2}$$

31.6 12)

$$\int_0^{\pi}\left(4\sigma\upsilon\nu 2x - \eta\mu 3x\right)dx = 4\int_0^{\pi}\sigma\upsilon\nu 2x\,dx + \int_0^{\pi}\left(-\eta\mu 3x\right)dx = 4\left[\frac{\eta\mu x}{2}\right]_0^{\pi} + \left[\frac{\sigma\upsilon\nu 3x}{3}\right]_0^{\pi} =$$

$$= 4\left(\frac{\eta\mu\pi}{2} - \frac{\eta\mu 0}{2}\right) + \left(\frac{\sigma\upsilon\nu 3\pi}{3} - \frac{\sigma\upsilon\nu 3\cdot 0}{3}\right) = 4(0-0) + \left(-\frac{1}{3} - \frac{1}{3}\right) = -\frac{2}{3}$$

31.6 13)

$$\int_0^{\pi}\left(\eta\mu\frac{x}{3} - \sigma\upsilon\nu\frac{x}{2}\right)dx = \int_0^{\pi}\eta\mu\frac{x}{3}dx - \int_0^{\pi}\sigma\upsilon\nu\frac{x}{2}dx = \left[-\frac{\sigma\upsilon\nu\frac{x}{3}}{\frac{1}{3}}\right]_0^{\pi} - \left[\frac{\eta\mu\frac{x}{2}}{\frac{1}{2}}\right]_0^{\pi} =$$

$$= -3\left[\sigma\upsilon\nu\frac{x}{3}\right]_0^{\pi} - 2\left[\eta\mu\frac{x}{2}\right]_0^{\pi} = -3\left[\sigma\upsilon\nu\frac{\pi}{3} - \sigma\upsilon\nu\frac{0}{3}\right]_0^{\pi} - 2\left[\eta\mu\frac{\pi}{2} - \eta\mu\frac{0}{2}\right]_0^{\pi} =$$

$$= -3\left(\frac{1}{2} - 1\right) - 2(1-0) = \frac{3}{2} - 2 = -\frac{1}{2}$$

31.6 14)

$$\int_{\frac{5}{2}}^{2}\left(e^{2x-4}+1\right)dx = \left[\frac{e^{2x-4}}{2} + x\right]_{\frac{5}{2}}^{2} = \frac{e^{2\cdot 2-4}}{2} + 2 - \left(\frac{e^{2\frac{5}{2}-4}}{2} + \frac{5}{2}\right) = \frac{1}{2} + 2 - \frac{e}{2} - \frac{5}{2} = -\frac{e}{2}$$

31.6 15)

$$\int_0^1 \left(6x^2 - e^{2x}\right)dx = \left[\cancel{6}^{2} \frac{x^3}{\cancel{3}} - \frac{e^{2x}}{2}\right]_0^1 = 2\cdot 1^3 - \frac{e^{2\cdot 1}}{2} - \left(2\cdot 0^3 - \frac{e^{2\cdot 0}}{2}\right) = 2 - \frac{e^2}{2} + \frac{1}{2} = \frac{5-e^2}{2}$$

31.6 16)

$$\int_0^2 \left(e^{3x} - 2x^2\right)dx = \int_0^2 e^{3x}dx - 2\int_0^2 x^2 dx = \left[\frac{e^{3x}}{3}\right]_0^2 - 2\left[\frac{x^3}{3}\right]_0^2 = \left(\frac{e^{3\cdot 2}}{3} - \frac{e^{3\cdot 0}}{3}\right) - 2\left(\frac{2^3}{3} - \frac{0^3}{3}\right) =$$

$$= \frac{e^6}{3} - \frac{1}{3} - \frac{16}{3} = \frac{e^6 - 17}{3}$$

31.6 17)

$$\int_0^1 \left(e^x - e^{-x}\right)dx = \left[e^x + e^{-x}\right]_0^1 = \left(e^1 + e^{-1}\right) - \left(e^0 + e^0\right) = e + \frac{1}{e} - 2 = \frac{e^2 - 2e + 1}{e}$$

31.6 18)

$$\int_{-1}^0 \left(3^{2x+1} + 2^{3x+2}\right)dx = \int_{-1}^0 3^{2x+1}dx + \int_{-1}^0 2^{3x+2}dx = \left[\frac{3^{2x+1}}{2\ln 3}\right]_{-1}^0 + \left[\frac{2^{3x+2}}{3\ln 2}\right]_{-1}^0 =$$

$$= \frac{1}{2\ln 3}\left[3^{2x+1}\right]_{-1}^0 + \frac{1}{3\ln 2}\left[2^{3x+2}\right]_{-1}^0 = \frac{1}{2\ln 3}\left[3^{2\cdot 0+1} - 3^{2\cdot(-1)+1}\right]_{-1}^0 + \frac{1}{3\ln 2}\left[2^{3\cdot 0+2} - 2^{3\cdot(-1)+2}\right]_{-1}^0 =$$

$$= \frac{1}{2\ln 3}\left(3 - \frac{1}{3}\right) + \frac{1}{3\ln 2}\left(2^2 - \frac{1}{2}\right) = \frac{8}{6\ln 3} + \frac{7}{6\ln 2} = \frac{8\ln 2 + 7\ln 3}{6\ln 3\cdot \ln 2}$$