31.56

α) $\int_0^{\frac{\pi}{2}} \frac{\eta\mu x}{\eta\mu x+\sigma\upsilon\nu x}dx = \int_0^{\frac{\pi}{2}} \frac{\sigma\upsilon\nu\left(\frac{\pi}{2}-x\right)}{\sigma\upsilon\nu\left(\frac{\pi}{2}-x\right)+\eta\mu\left(\frac{\pi}{2}-x\right)}dx =$

θέτουμε $y=\frac{\pi}{2}-x \Rightarrow dy=-dx$

για $x=0 \Rightarrow y=\frac{\pi}{2}$

για $x=\frac{\pi}{2} \Rightarrow y=0$

$= -\int_{\frac{\pi}{2}}^0 \frac{\sigma\upsilon\nu y}{\sigma\upsilon\nu y+\eta\mu y}dy = \int_0^{\frac{\pi}{2}} \frac{\sigma\upsilon\nu x}{\sigma\upsilon\nu x+\eta\mu x}dx$

β) Έστω $I=\int_0^{\frac{\pi}{2}} \frac{\eta\mu x}{\eta\mu x+\sigma\upsilon\nu x}dx = \int_0^{\frac{\pi}{2}} \frac{\sigma\upsilon\nu x}{\eta\mu x+\sigma\upsilon\nu x}dx$. Τότε

$$\left.\begin{aligned} I&=\int_0^{\frac{\pi}{2}} \frac{\eta\mu x}{\eta\mu x+\sigma\upsilon\nu x}dx \\ I&=\int_0^{\frac{\pi}{2}} \frac{\sigma\upsilon\nu x}{\eta\mu x+\sigma\upsilon\nu x}dx \end{aligned}\right\} \overset{\text{προσθέτουμε κατά μέλη}}{\Rightarrow} 2I=\int_0^{\frac{\pi}{2}} \frac{\eta\mu x}{\eta\mu x+\sigma\upsilon\nu x}dx+\int_0^{\frac{\pi}{2}} \frac{\sigma\upsilon\nu x}{\eta\mu x+\sigma\upsilon\nu x}dx \Rightarrow$$

$$\Rightarrow 2I=\int_0^{\frac{\pi}{2}} \frac{\eta\mu x}{\eta\mu x+\sigma\upsilon\nu x}+\frac{\sigma\upsilon\nu x}{\eta\mu x+\sigma\upsilon\nu x}dx \Rightarrow 2I=\int_0^{\frac{\pi}{2}} \frac{\cancel{\eta\mu x+\sigma\upsilon\nu x}}{\cancel{\eta\mu x+\sigma\upsilon\nu x}}dx \Rightarrow$$

$$\Rightarrow 2I=1\left(\frac{\pi}{2}-0\right) \Rightarrow 2I=\frac{\pi}{2} \Rightarrow I=\frac{\pi}{4}$$