

Έχουμε ισοδύναμα

$$\boxed{\int_a^\beta f(t) dt + \int_a^{-\beta} f(t) dt = 2 \int_a^0 f(t) dt} \Leftrightarrow$$

$$\Leftrightarrow \cancel{\int_a^0 f(t) dt} + \int_0^\beta f(t) dt + \cancel{\int_a^0 f(t) dt} + \int_0^{-\beta} f(t) dt = 2 \cancel{\int_a^0 f(t) dt} \Leftrightarrow$$

$$\Leftrightarrow \int_0^\beta f(t) dt + \int_0^{-\beta} f(t) dt = 0 \quad \underline{\text{που ισχύει}} \text{ διότι}$$

$$\int_0^{-\beta} f(t) dt \stackrel{\substack{\text{θέτουμε } y=-t \Rightarrow dy=-dt \\ \text{για } x=0 \Rightarrow y=0 \\ \text{για } x=-\beta \Rightarrow y=\beta}}{=} - \int_0^\beta f(-t) dt \stackrel{f:\text{άρτια}}{=} - \int_0^\beta f(t) dt$$