

Είναι

$$\int_{-a}^a f(x) dx = \int_{-a}^0 f(x) dx + \int_0^a f(x) dx$$

όμως

$$\boxed{\int_{-a}^0 f(x) dx} \overset{\substack{\text{θέτουμε } y=-x \Rightarrow dy=-dx \\ \text{για } x=-a \Rightarrow y=a \\ \text{για } x=0 \Rightarrow y=0}}{=} - \int_a^0 f(-y) dy \overset{\text{f:άρτια}}{=} \int_0^a f(y) dy = \boxed{\int_0^a f(x) dx}$$

$$\Rightarrow \int_{-a}^a f(x) dx = \int_0^a f(x) dx + \int_0^a f(x) dx \Rightarrow \int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$$