

Έστω $I = \int_a^b x f(x) dx$. Τότε

$$I = \int_a^b x f(x) dx \stackrel{f(x)=f(a+b-x)}{=} \int_a^b x f(a+b-x) dx \stackrel{\substack{\text{θέτουμε } y=a+b-x \Rightarrow x=a+b-y \\ \Rightarrow dx=-dy \\ \text{για } x=a \Rightarrow y=a+b-a=b \\ \text{για } x=b \Rightarrow y=a+b-b=a}}{=} - \int_b^a (a+b-y) f(y) dy =$$

$$= \int_a^b (a+b) f(y) - y f(y) dy = \int_a^b (a+b) f(y) dy - \int_a^b y f(y) dy = (a+b) \int_a^b f(y) dy - I$$

Επομένως

$$I = (a+b) \int_a^b f(y) dy - I \Rightarrow 2I = (a+b) \int_a^b f(y) dy \Rightarrow I = \frac{a+b}{2} \int_a^b f(y) dy$$