

$$\begin{aligned}
 & \text{θέτουμε } y = \frac{\pi}{2} - x \Rightarrow dy = -dx \\
 & \text{για } x=0 \Rightarrow y = \frac{\pi}{2} - 0 = \frac{\pi}{2} \\
 & \text{για } x = \frac{\pi}{2} \Rightarrow y = \frac{\pi}{2} - \frac{\pi}{2} = 0 \\
 & \boxed{\int_0^{\frac{\pi}{2}} f(\sin x) dx} = \int_0^{\frac{\pi}{2}} f\left(\sin\left(\frac{\pi}{2} - x\right)\right) dx = - \int_{\frac{\pi}{2}}^0 f(\sin y) dy = \\
 & = \int_0^{\frac{\pi}{2}} f(\sin y) dy = \boxed{\int_0^{\frac{\pi}{2}} f(\sin x) dx}
 \end{aligned}$$