

Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Β

31.42 1)

$$\int_0^2 \frac{2x+1}{\sqrt{x^2+x+2}} dx \quad \begin{array}{l} \text{θέτουμε } y=x^2+x+2 \Rightarrow dy=(2x+1)dx \\ \text{για } x=0 \Rightarrow y=2 \\ \text{για } x=2 \Rightarrow y=8 \end{array} = \int_2^8 \frac{1}{\sqrt{y}} dy = \left[2\sqrt{y} \right]_2^8 = 2\sqrt{8} - 2\sqrt{2} \stackrel{\sqrt{8}=2\sqrt{2}}{=} = 2\sqrt{2}$$

31.42 2)

$$\int_{16}^{49} \sqrt{\frac{\sqrt{x}-3}{x}} dx = \int_{16}^{49} \frac{\sqrt{\sqrt{x}-3}}{\sqrt{x}} dx \quad \begin{array}{l} \text{θέτουμε } y=\sqrt{x} \Rightarrow dy=\frac{1}{2\sqrt{x}} dx \Rightarrow \frac{1}{\sqrt{x}} dx=2dy \\ \text{για } x=16 \Rightarrow y=4 \\ \text{για } x=49 \Rightarrow y=7 \end{array} = \int_4^7 \sqrt{y-3} \cdot 2dy =$$

$$= 2 \int_4^7 \sqrt{y-3} dy \quad \begin{array}{l} \text{θέτουμε } u=y-3 \Rightarrow du=dy \\ \text{για } y=4 \Rightarrow u=1 \\ \text{για } y=7 \Rightarrow u=4 \end{array} = 2 \int_1^4 \sqrt{u} du = 2 \left[\frac{u^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^4 = 2 \cdot \frac{2}{3} \left[\sqrt{u^3} \right]_1^4 = \frac{4}{3} \left(\sqrt{4^3} - \sqrt{1^3} \right) = \frac{28}{3}$$

31.42 3)

$$\int_1^2 \frac{1}{\sqrt{x+2} + \sqrt{x-1}} dx \quad \begin{array}{l} \text{πολλαπλασιάζουμε} \\ \text{με την συζυγή} \end{array} = \int_1^2 \frac{\sqrt{x+2} - \sqrt{x-1}}{(\sqrt{x+2} + \sqrt{x-1})(\sqrt{x+2} - \sqrt{x-1})} dx =$$

$$= \int_1^2 \frac{\sqrt{x+2} - \sqrt{x-1}}{\sqrt{x+2}^2 - \sqrt{x-1}^2} dx = \int_1^2 \frac{\sqrt{x+2} - \sqrt{x-1}}{\cancel{x}+2 - \cancel{x}+1} dx = \frac{1}{3} \int_1^2 (\sqrt{x+2} - \sqrt{x-1}) dx =$$

$$= \frac{1}{3} \left[\frac{(x+2)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{(x-1)^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^2 = \frac{1}{3} \cdot \frac{2}{3} \left[\sqrt{x+2}^3 - \sqrt{x-1}^3 \right]_1^2 =$$

$$= \frac{2}{9} \left[\left(\sqrt{2+2}^3 - \sqrt{2-1}^3 \right) - \left(\sqrt{1+2}^3 - \sqrt{1-1}^3 \right) \right] = \frac{2}{9} (7 - 3\sqrt{3}) = \frac{14 - 6\sqrt{3}}{9}$$