

Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Β

31.41 1)

$$\begin{aligned} \int_e^{e^2} \frac{\ln x + 1}{x \ln x} dx & \stackrel{\substack{\text{θέτουμε } y = \ln x \Rightarrow dy = \frac{1}{x} dx \\ \text{για } x = e \Rightarrow y = \ln e = 1 \\ \text{για } x = e^2 \Rightarrow y = \ln e^2 = 2}}{=} \int_1^2 \frac{y + 1}{y} dy = \int_1^2 \frac{y}{y} + \frac{1}{y} dy = \int_1^2 \left(1 + \frac{1}{y} \right) dy = \\ & = \left[y + \ln |y| \right]_1^2 = 2 + \ln 2 - (1 + \ln 1) = 1 + \ln 2 \end{aligned}$$

31.41 2)

$$\begin{aligned} \int_e^{e^2} \frac{\ln^2 x - 1}{x \ln x} dx & \stackrel{\substack{\text{θέτουμε } y = \ln x \Rightarrow dy = \frac{1}{x} dx \\ \text{για } x = e \Rightarrow y = \ln e = 1 \\ \text{για } x = e^2 \Rightarrow y = \ln e^2 = 2}}{=} \int_1^2 \frac{y^2 - 1}{y} dy = \int_1^2 \frac{y^2}{y} - \frac{1}{y} dy = \int_1^2 \left(y - \frac{1}{y} \right) dy = \\ & = \left[\frac{y^2}{2} - \ln |y| \right]_1^2 = \frac{2^2}{2} - \ln 2 - \left(\frac{1^2}{2} - \ln 1 \right) = \frac{3}{2} - \ln 2 \end{aligned}$$

31.41 3)

$$\int_1^e \frac{1 - \ln x}{x^2} dx = \int_1^e \left(\frac{\ln x}{x} \right)' dx = \left[\frac{\ln x}{x} \right]_1^e = \frac{\ln e}{e} - \frac{\ln 1}{1} = \frac{1}{e}$$