

Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Β

31.37 1)

$$\int_0^1 x^2 (1-x)^7 dx \stackrel{\substack{\text{θέτουμε } y=1-x \Rightarrow x=1-y \Rightarrow \\ \Rightarrow dy=-dx \\ \text{για } x=0 \Rightarrow y=1-0=1 \\ \text{για } x=1 \Rightarrow y=1-1=0}}{=} - \int_1^0 (1-y)^2 y^7 dy = \int_0^1 (1-2y+y^2) y^7 dy =$$

$$= \int_0^1 y^7 - 2y^8 + y^9 dy = \left[\frac{y^8}{8} - \frac{2y^9}{9} + \frac{y^{10}}{10} \right]_0^1 = \frac{1^8}{8} - \frac{2 \cdot 1^9}{9} + \frac{1^{10}}{10} = \frac{1}{8} - \frac{2}{9} + \frac{1}{10} = \frac{1}{360}$$

31.37 2)

$$\boxed{\int_0^1 x^k (1-x)^l dx} \stackrel{\substack{\text{θέτουμε } y=1-x \Rightarrow x=1-y \Rightarrow \\ \Rightarrow dy=-dx \\ \text{για } x=0 \Rightarrow y=1-0=1 \\ \text{για } x=1 \Rightarrow y=1-1=0}}{=} - \int_1^0 (1-y)^k y^l dy = \int_0^1 (1-y)^k y^l dy = \boxed{\int_0^1 (1-x)^k x^l dx}$$

31.37 3)

$$\int_1^3 x^2 (2-x)^7 dx \stackrel{\substack{\text{θέτουμε } y=2-x \Rightarrow x=2-y \Rightarrow \\ \Rightarrow dy=-dx \\ \text{για } x=1 \Rightarrow y=2-1=1 \\ \text{για } x=3 \Rightarrow y=2-3=-1}}{=} - \int_1^{-1} (2-y)^2 y^7 dy = \int_{-1}^1 (4-4y+y^2) y^7 dy =$$

$$= \int_{-1}^1 4y^7 - 4y^8 + y^9 dy = \left[\frac{4y^8}{8} - \frac{4y^9}{9} + \frac{y^{10}}{10} \right]_{-1}^1 =$$

$$= \frac{1^8}{2} - \frac{4 \cdot 1^9}{9} + \frac{1^{10}}{10} - \left[\frac{(-1)^8}{2} - \frac{4 \cdot (-1)^9}{9} + \frac{(-1)^{10}}{10} \right] = \frac{1}{2} - \frac{4}{9} + \frac{1}{10} - \left(\frac{1}{2} + \frac{4}{9} + \frac{1}{10} \right) =$$

$$= \frac{1}{2} - \frac{4}{9} + \frac{1}{10} - \frac{1}{2} - \frac{4}{9} - \frac{1}{10} = -\frac{8}{9}$$