

Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Β

31.36 1)

$$\int_0^a f(a-x) dx = \int_a^0 f(y) dy = \int_0^a f(y) dy = \int_0^a f(x) dx$$

$\theta\acute{\epsilon}τουμε y=a-x \Rightarrow dy=-dx$
 $\gamma\acute{\iota}α x=0 \Rightarrow y=a$
 $\gamma\acute{\iota}α x=a \Rightarrow y=0$

31.36 2)

$$(\beta-\alpha) \int_0^1 f[\alpha+(\beta-\alpha)x] dx = \int_0^1 f[\alpha+(\beta-\alpha)x] (\beta-\alpha) dx$$

$$\theta\acute{\epsilon}τουμε y=\alpha+(\beta-\alpha)x \Rightarrow dy=(\beta-\alpha)dx$$

$\gamma\acute{\iota}α x=0 \Rightarrow y=\alpha+(\beta-\alpha) \cdot 0=\alpha$
 $\gamma\acute{\iota}α x=1 \Rightarrow y=\alpha+(\beta-\alpha) \cdot 1=\beta$

$$= \int_{\alpha}^{\beta} f(y) dy$$

31.36 3)

$$11 \int_0^1 f(11x-5) dx = \int_{-5}^6 f(y) \frac{1}{11} dy = \int_{-5}^6 f(y) dy$$

$\theta\acute{\epsilon}τουμε y=11x-5 \Rightarrow dy=11dx \Rightarrow dx=\frac{1}{11}dy$
 $\gamma\acute{\iota}α x=0 \Rightarrow y=11 \cdot 0-5=-5$
 $\gamma\acute{\iota}α x=1 \Rightarrow y=11 \cdot 1-5=6$

31.36 4)

$$\int_{\alpha+c}^{\beta+c} f(x-c) dx = \int_{\alpha}^{\beta} f(y) dy$$

$\theta\acute{\epsilon}τουμε y=x-c \Rightarrow dy=dx$
 $\gamma\acute{\iota}α x=\alpha+c \Rightarrow y=\alpha+c-c=\alpha$
 $\gamma\acute{\iota}α x=\beta+c \Rightarrow y=\beta+c-c=\beta$

ΚΑΙ

$$\frac{1}{c} \int_{c\alpha}^{c\beta} f\left(\frac{x}{c}\right) dx = \int_{\alpha}^{\beta} f(y) dy$$

$\theta\acute{\epsilon}τουμε y=\frac{x}{c} \Rightarrow dy=\frac{1}{c}dx \Rightarrow dx=c dy$
 $\gamma\acute{\iota}α x=c\alpha \Rightarrow y=\frac{c\alpha}{c}=\alpha$
 $\gamma\acute{\iota}α x=c\beta \Rightarrow y=\frac{c\beta}{c}=\beta$

$$\text{Επομένως } \int_{\alpha}^{\beta} f(x) dx = \int_{\alpha+c}^{\beta+c} f(x-c) dx = \frac{1}{c} \int_{c\alpha}^{c\beta} f\left(\frac{x}{c}\right) dx$$

31.36 5)

$$\int_0^a f(-x) dx = - \int_0^{-a} f(y) dy = \int_{-a}^0 f(y) dy = \int_{-a}^0 f(x) dx$$

$\theta\acute{\epsilon}τουμε y=-x \Rightarrow dy=-dx$
 $\gamma\acute{\iota}α x=0 \Rightarrow y=0$
 $\gamma\acute{\iota}α x=a \Rightarrow y=-a$

31.36 6)

$$\int_0^a [f(x) + f(-x)] dx = \int_0^a f(x) dx + \int_0^a f(-x) dx =$$

$$= \int_0^a f(x) dx - \int_0^{-a} f(y) dy = \int_0^a f(x) dx - \int_0^{-a} f(y) dy = \int_0^a f(x) dx + \int_{-a}^0 f(y) dy =$$

$\theta\acute{\epsilon}τουμε y=-x \Rightarrow dy=-dx$
 $\gamma\acute{\iota}α x=0 \Rightarrow y=0$
 $\gamma\acute{\iota}α x=a \Rightarrow y=-a$

$$= \int_0^a f(x) dx + \int_{-a}^0 f(x) dx = \int_{-a}^a f(x) dx$$

31.36 7)

$$\int_{c-\beta}^{c-\alpha} f(c-x) dx \stackrel{\substack{\text{θέτουμε } y=c-x \Rightarrow dy=-dx \\ \text{για } x=c-\beta \Rightarrow y=c-\beta+\beta=\beta \\ \text{για } x=c-\alpha \Rightarrow y=c-\alpha+\alpha=\alpha}}{=} -\int_{\beta}^{\alpha} f(y) dy = \int_{\alpha}^{\beta} f(y) dy = \int_{\alpha}^{\beta} f(x) dx$$

31.36 8)

$$\alpha) \quad \int_{\alpha}^{\beta} f(\alpha+\beta-x) dx \stackrel{\substack{\text{θέτουμε } y=\alpha+\beta-x \Rightarrow dy=-dx \\ \text{για } x=\alpha \Rightarrow y=\alpha+\beta-\alpha=\beta \\ \text{για } x=\beta \Rightarrow y=\alpha+\beta-\beta=\alpha}}{=} -\int_{\beta}^{\alpha} f(y) dy = \int_{\alpha}^{\beta} f(y) dy = \int_{\alpha}^{\beta} f(x) dx$$

$$\beta) \quad \int_0^{\frac{\pi}{2}} \eta \mu^7 x \, dx \stackrel{\alpha) \text{ ερώτημα}}{=} \int_0^{\frac{\pi}{2}} \eta \mu^7 \left(0 + \frac{\pi}{2} - x\right) dx = \int_0^{\frac{\pi}{2}} \eta \mu^7 \left(\frac{\pi}{2} - x\right) dx = \int_0^{\frac{\pi}{2}} \sigma \upsilon \nu^7 x \, dx$$