Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Β

31.35 1)

$$\boxed{\int_0^1\left[x^2f''(x)-2f(x)\right]dx}=\int_0^1 x^2f''(x)dx-\int_0^1 2f(x)dx \overset{\text{παραγοντική ολοκλήρωση}}{=}$$

$$=\left[x^2f'(x)\right]_0^1-\int_0^1\left(x^2\right)'f'(x)dx-\int_0^1 2f(x)dx=f'(1)-\int_0^1 2xf'(x)dx-\int_0^1 2f(x)dx=$$

$$\overset{\text{παραγοντική ολοκλήρωση}}{=} f'(1)-\left(\left[2xf(x)\right]_0^1-\int_0^1(2x)'f(x)dx\right)-\int_0^1 2f(x)dx=$$

$$=f'(1)-\left(2f(1)-\int_0^1 2f(x)dx\right)-\int_0^1 2f(x)dx=$$

$$=f'(1)-2f(1)+\cancel{\int_0^1 2f(x)dx}-\cancel{\int_0^1 2f(x)dx}=\boxed{f'(1)-2f(1)}$$

31.35 2)

$$\boxed{\int_\alpha^\beta e^x\left(f''(x)-f(x)\right)dx}=\int_\alpha^\beta e^xf''(x)-e^xf(x)dx$$

$$=\int_\alpha^\beta e^xf''(x)dx-\int_\alpha^\beta e^xf(x)dx \overset{\text{παραγοντική ολοκλήρωση}}{=}$$

$$=\left[e^xf'(x)\right]_\alpha^\beta-\int_\alpha^\beta\left(e^x\right)'f'(x)dx-\int_\alpha^\beta e^xf(x)=$$

$$=e^\beta f'(\beta)-e^\alpha f'(\alpha)-\int_\alpha^\beta e^xf'(x)dx-\int_\alpha^\beta e^xf(x)dx \overset{\text{παραγοντική ολοκλήρωση}}{=}$$

$$=e^\beta f'(\beta)-e^\alpha f'(\alpha)-\left(\left[e^xf(x)\right]_\alpha^\beta-\int_\alpha^\beta\left(e^x\right)'f(x)dx\right)-\int_\alpha^\beta e^xf(x)dx=$$

$$=e^\beta f'(\beta)-e^\alpha f'(\alpha)-\left(e^\beta f(\beta)-e^\alpha f(\alpha)-\int_\alpha^\beta e^xf(x)dx\right)-\int_\alpha^\beta e^xf(x)dx=$$

$$=e^\beta f'(\beta)-e^\alpha f'(\alpha)-e^\beta f(\beta)+e^\alpha f(\alpha)+\cancel{\int_\alpha^\beta e^xf(x)dx}-\cancel{\int_\alpha^\beta e^xf(x)dx}=$$

$$=\boxed{e^\beta\left[f'(\beta)-f(\beta)\right]-e^\alpha\left[f'(\alpha)-f(\alpha)\right]}$$

31.35 3)

$$\boxed{\int_0^{\frac{\pi}{2}}\left[f(x)+f''(x)\right]\text{συν}x\,dx}=\int_0^{\frac{\pi}{2}}\left[f(x)\text{συν}x+f''(x)\text{συν}x\right]dx=$$

$$=\int_0^{\frac{\pi}{2}}f(x)\text{συν}x\,dx+\int_0^{\frac{\pi}{2}}f''(x)\text{συν}x\,dx \overset{\text{παραγοντική ολοκλήρωση}}{=}$$

$$=\int_0^{\frac{\pi}{2}}f(x)\text{συν}x\,dx+\left[f'(x)\text{συν}x\right]_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}f'(x)(\text{συν}x)'\,dx=$$

$$=\int_0^{\frac{\pi}{2}} f(x)\sigma\upsilon\nu x dx + \cancel{f'\left(\frac{\pi}{2}\right)\sigma\upsilon\nu\frac{\pi}{2}} - f'(0)\sigma\upsilon\nu 0 - \int_0^{\frac{\pi}{2}} f'(x)(-\eta\mu x)dx =$$

$$=\int_0^{\frac{\pi}{2}} f(x)\sigma\upsilon\nu x dx - f'(0) + \int_0^{\frac{\pi}{2}} f'(x)\eta\mu x dx \overset{\text{παραγοντική ολοκλήρωση}}{=}$$

$$=\int_0^{\frac{\pi}{2}} f(x)\sigma\upsilon\nu x dx - f'(0) + \left[f(x)\eta\mu x\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} f(x)(\eta\mu x)' dx =$$

$$=\cancel{\int_0^{\frac{\pi}{2}} f(x)\sigma\upsilon\nu x dx} - f'(0) + f\left(\frac{\pi}{2}\right)\eta\mu\frac{\pi}{2} - f(0)\eta\mu 0 - \cancel{\int_0^{\frac{\pi}{2}} f(x)\sigma\upsilon\nu x dx} =$$

$$=\boxed{f\left(\frac{\pi}{2}\right) - f'(0)}$$

31.35 4)

$$\boxed{\int_1^e xf''(x)dx} \overset{\text{παραγοντική ολοκλήρωση}}{=} \left[xf'(x)\right]_1^e - \int_1^e (x)' f'(x)dx =$$

$$= ef'(e) - f'(1) - \int_1^e f'(x)dx = ef'(e) - f'(1) - \left[f(x)\right]_1^e =$$

$$= ef'(e) - f'(1) - \left[f(e) - f(1)\right] = f'(e) - f'(1) - f(e) + f(1) \overset{f(1)=f(e)}{=} \boxed{f'(e) - f'(1)}$$

31.35 5)

Επειδή η g παρουσιάζει ακρότατο στο $x_0 = 0$ σύμφωνα με το Θεώρημα Fermat θα είναι $\boxed{g'(0) = 0}$

Επειδή η C_g εφάπτεται στον άξονα x'x στο 2 θα είναι $\boxed{g(2) = 0}$ και $\boxed{g'(2) = 0}$

Οπότε

$$\int_0^2 \frac{1}{e^x}\left[g(x) - g''(x)\right]dx = \int_0^2 e^{-x}g(x)dx - \int_0^2 e^{-x}g''(x)dx \overset{\text{παραγοντική ολοκλήρωση}}{=}$$

$$= \int_0^2 e^{-x}g(x)dx - \left(\left[e^{-x}g'(x)\right]_0^2 - \int_0^2 \left(e^{-x}\right)' g'(x)dx\right) =$$

$$= \int_0^2 e^{-x}g(x)dx - \left(e^{-2}g'(2) - e^0 g'(0) - \int_0^2 -e^{-x}g'(x)dx\right) =$$

$$\overset{\substack{g(2)=0\\ g'(2)=0}}{=} \int_0^2 e^{-x}g(x)dx - \int_0^2 e^{-x}g'(x)dx \overset{\text{παραγοντική ολοκλήρωση}}{=}$$

$$= \int_0^2 e^{-x}g(x)dx - \left(\left[e^{-x}g(x)\right]_0^2 - \int_0^2 \left(e^{-x}\right)' g(x)dx\right) =$$

$$= \int_0^2 e^{-x}g(x)dx - \left(e^{-2}g(2) - e^0 g(0) - \int_0^2 -e^{-x}g(x)dx\right) =$$

$$\overset{g(2)=0}{=} \cancel{\int_0^2 e^{-x}g(x)dx} + g(0) - \cancel{\int_0^2 e^{-x}g(x)dx} = \boxed{g(0)}$$

31.35 6)

$$\int_1^2 \left[f(x)+f'(x)\right]e^x dx = 1 \Rightarrow \int_1^2 f(x)e^x + f'(x)e^x dx = 1 \Rightarrow$$

$$\Rightarrow \int_1^2 f(x)e^x dx + \int_1^2 f'(x)e^x dx = 1 \overset{\text{παραγοντική ολοκλήρωση}}{\Rightarrow}$$

$$\Rightarrow \int_1^2 f(x)e^x dx + \left[f(x)e^x\right]_1^2 - \int_1^2 f(x)\left(e^x\right)' dx = 1 \Rightarrow$$

$$\Rightarrow \cancel{\int_1^2 f(x)e^x dx} + f(2)e^2 - f(1)e^1 - \cancel{\int_1^2 f(x)e^x dx} = 1 \Rightarrow$$

$$\overset{f(1)=\frac{1}{e}}{\Rightarrow} f(2)e^2 - \frac{1}{\cancel{e}}\cancel{e} = 1 \Rightarrow f(2)e^2 - 1 = 1 \Rightarrow f(2)e^2 = 2 \Rightarrow f(2) = \frac{2}{e^2} \Rightarrow \boxed{f(2) = 2e^{-2}}$$

31.35 7)

$$\int_0^{\frac{\pi}{2}} \left[f''(x)+f(x)\right]\eta\mu x\, dx = 1 \Rightarrow \int_0^{\frac{\pi}{2}} f''(x)\eta\mu x + f(x)\eta\mu x\, dx = 1 \Rightarrow$$

$$\Rightarrow \int_0^{\frac{\pi}{2}} f''(x)\eta\mu x\, dx + \int_0^{\frac{\pi}{2}} f(x)\eta\mu x\, dx = 1 \overset{\text{παραγοντική ολοκλήρωση}}{\Rightarrow}$$

$$\Rightarrow \left[f'(x)\eta\mu x\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} f'(x)\left(\eta\mu x\right)' dx + \int_0^{\frac{\pi}{2}} f(x)\eta\mu x dx = 1 \Rightarrow$$

$$\Rightarrow f'\left(\frac{\pi}{2}\right)\eta\mu\frac{\pi}{2} - \cancel{f'(0)\eta\mu 0} - \int_0^{\frac{\pi}{2}} f'(x)\sigma\upsilon\nu x\, dx + \int_0^{\frac{\pi}{2}} f(x)\eta\mu x dx = 1 \Rightarrow$$

$$\overset{f'\left(\frac{\pi}{2}\right)=3}{\Rightarrow} 3 - \int_0^{\frac{\pi}{2}} f'(x)\sigma\upsilon\nu x\, dx + \int_0^{\frac{\pi}{2}} f(x)\eta\mu x dx = 1 \overset{\text{παραγοντική ολοκλήρωση}}{\Rightarrow}$$

$$\Rightarrow 3 - \left(\left[f(x)\sigma\upsilon\nu x\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} f(x)\left(\sigma\upsilon\nu x\right)' dx\right) + \int_0^{\frac{\pi}{2}} f(x)\eta\mu x dx = 1 \Rightarrow$$

$$\Rightarrow 3 - \left(\cancel{f\left(\frac{\pi}{2}\right)\sigma\upsilon\nu\frac{\pi}{2}} - f(0)\sigma\upsilon\nu 0 - \int_0^{\frac{\pi}{2}} f(x)\left(-\eta\mu x\right) dx\right) + \int_0^{\frac{\pi}{2}} f(x)\eta\mu x dx = 1 \Rightarrow$$

$$\Rightarrow 3 + f(0) - \cancel{\int_0^{\frac{\pi}{2}} f(x)\eta\mu x\, dx} + \cancel{\int_0^{\frac{\pi}{2}} f(x)\eta\mu x dx} = 1 \Rightarrow 3 + f(0) = 1 \Rightarrow \boxed{f(0) = -2}$$

31.35 8)

$$\boxed{\int_1^3 \left[(x-1)f''(x)+f'(x)\right]dx} = \int_1^3 (x-1)f''(x)dx + \int_1^3 f'(x)dx =$$

$$\overset{\text{παραγοντική ολοκλήρωση}}{=} \left[(x-1)f'(x)\right]_1^3 - \int_1^3 (x-1)' f'(x)dx + \int_1^3 f'(x)dx =$$

$$= (3-1)f'(3) - (1-1)f'(1) - \cancel{\int_1^3 f'(x)dx} + \cancel{\int_1^3 f'(x)dx} = \boxed{2f'(3)}$$