Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Β

31.33 1)

θέτουμε $y=x^2 \Rightarrow dy=2xdx \Rightarrow xdx=\frac{1}{2}dy$

για $x=0 \Rightarrow y=0^2=0$

για $x=1 \Rightarrow y=1^2=1$

$$\int_0^1 x^3 e^{x^2} dx = \int_0^1 x^2 e^{x^2} x dx = \int_0^1 y e^y \frac{1}{2} dx = \frac{1}{2}\int_0^1 y\left(e^y\right)' dx =$$

$$= \frac{1}{2}\left(\left[ye^y\right]_0^1 - \int_0^1 (y)' e^y dx\right) = \frac{1}{2}\left(e - \int_0^1 e^y dx\right) = \frac{1}{2}e - \frac{1}{2}\left[e^y\right]_0^1 = \frac{1}{2}e - \frac{1}{2}(e-1) =$$

$$= \cancel{\frac{1}{2}e} - \cancel{\frac{1}{2}e} + \frac{1}{2} = \frac{1}{2}$$

31.33 2)

θέτουμε $y=\sqrt{x} \Rightarrow dy=\frac{1}{2\sqrt{x}}dx \Rightarrow dy=\frac{1}{2y}dx \Rightarrow dx=2ydy$

για $x=0 \Rightarrow y=\sqrt{0}=0$

για $x=1 \Rightarrow y=\sqrt{1}=1$

$$\int_0^1 e^{\sqrt{x}} dx = \int_0^1 e^y 2ydy = 2\int_0^1 y\left(e^y\right)' dy =$$

$$= 2\left(\left[ye^y\right]_0^1 - \int_0^1 (y)' e^y dx\right) = 2\left(e - \int_0^1 e^y dx\right) = 2\left(e - \left[e^x\right]_0^1\right) = 2\left[e-(e-1)\right] = 2$$

31.33 3)

θέτουμε $y=\sqrt{x} \Rightarrow dy=\frac{1}{2\sqrt{x}}dx \Rightarrow dy=\frac{1}{2y}dx \Rightarrow dx=2ydy$

για $x=0 \Rightarrow y=\sqrt{0}=0$

για $x=\pi^2 \Rightarrow y=\sqrt{\pi^2}=\pi$

$$\int_0^{\pi^2} συν\sqrt{x}\, dx = \int_0^{\pi} συνy \cdot 2ydy = 2\int_0^{\pi} yσυνydy =$$

$$= 2\int_0^{\pi} y(ημy)' dy = 2\left(\left[yημy\right]_0^{\pi} - \int_0^{\pi} (y)' ημydy\right) = 2\left(0 - \int_0^{\pi} ημydy\right) =$$

$$= -2\int_0^{\pi} ημydy = -2\left[-συνy\right]_0^{\pi} = -2(-συνπ + συν0) = -2\left[-(-1)+1\right] = -4$$

31.33 4)

θέτουμε $y=\sqrt{x} \Rightarrow dy=\frac{1}{2\sqrt{x}}dx \Rightarrow dy=\frac{1}{2y}dx \Rightarrow dx=2ydy$

για $x=0 \Rightarrow y=\sqrt{0}=0$

για $x=\frac{\pi^2}{4} \Rightarrow y=\sqrt{\frac{\pi^2}{4}}=\frac{\pi}{2}$

$$\int_0^{\frac{\pi^2}{4}} ημ\sqrt{x}\, dx = \int_0^{\frac{\pi}{2}} ημy \cdot 2ydy = 2\int_0^{\frac{\pi}{2}} yημydy =$$

$$= 2\int_0^{\frac{\pi}{2}} y(-συνy)' dy = 2\left(\left[-yσυνy\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} (y)'(-συνy)dy\right) =$$

$$= 2\left(0 + \int_0^{\frac{\pi}{2}} συνydy\right) = 2\int_0^{\frac{\pi}{2}} συνydy = 2\left[ημx\right]_0^{\frac{\pi}{2}} = 2\left(ημ\frac{\pi}{2} - ημ0\right) = 2$$