Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Β

31.32 1)

α) $$\int_{-3}^{3}|x|dx=\int_{-3}^{0}|x|dx+\int_{0}^{3}|x|dx=\int_{-3}^{0}-xdx+\int_{0}^{3}xdx=\left[-\frac{x^2}{2}\right]_{-3}^{0}+\left[\frac{x^2}{2}\right]_{0}^{3}=$$

$$=-\frac{1}{2}\left[x^2\right]_{-3}^{0}+\frac{1}{2}\left[x^2\right]_{0}^{3}=-\frac{1}{2}(0-9)+\frac{1}{2}(9-0)=9$$

β) $$\int_{-2}^{3}\left(|x-2|+|x+1|\right)dx=\int_{-2}^{3}|x-2|dx+\int_{-2}^{3}|x+1|dx=$$

$$=\int_{-2}^{2}|x-2|dx+\int_{2}^{3}|x-2|dx+\int_{-2}^{-1}|x+1|dx+\int_{-1}^{3}|x+1|dx=$$

$$=\int_{-2}^{2}(-x+2)dx+\int_{2}^{3}(x-2)dx+\int_{-2}^{-1}(-x-1)dx+\int_{-1}^{3}(x+1)dx=$$

$$=\left[-\frac{x^2}{2}+2x\right]_{-2}^{2}+\left[\frac{x^2}{2}-2x\right]_{2}^{3}+\left[-\frac{x^2}{2}-x\right]_{-2}^{-1}+\left[\frac{x^2}{2}+x\right]_{-1}^{3}=\cdots=17$$

γ) $$\int_{-3}^{1}\left|x^2+2x\right|dx\overset{\substack{x^2+2x>0\Leftrightarrow x<-2\ ή\ x>0\\ x^2+x<0\Leftrightarrow -2<x<0}}{=}\int_{-3}^{-2}\left|x^2+2x\right|dx+\int_{-2}^{0}\left|x^2+2x\right|dx+\int_{0}^{1}\left|x^2+2x\right|dx=$$

$$=\int_{-3}^{-2}\left(x^2+2x\right)dx+\int_{-2}^{0}\left(-x^2-2x\right)dx+\int_{0}^{1}\left(x^2+2x\right)dx=$$

$$=\left[\frac{x^3}{3}+x^2\right]_{-3}^{-2}+\left[-\frac{x^3}{3}-x^2\right]_{-2}^{0}+\left[\frac{x^3}{3}+x^2\right]_{0}^{1}=$$

$$=\left(\frac{-8}{3}+4\right)-\left(\frac{-27}{3}+9\right)+0-\left(-\frac{-8}{3}-4\right)+\left(\frac{1}{3}+1\right)-0=4$$

31.32 2)

$$\int_{-2}^{4}|x|dx=\int_{-2}^{0}|x|dx+\int_{0}^{4}|x|dx=\int_{-2}^{0}-xdx+\int_{0}^{4}xdx=\left[-\frac{x^2}{2}\right]_{-2}^{0}+\left[\frac{x^2}{2}\right]_{0}^{4}=$$

$$=-\frac{1}{2}\left[x^2\right]_{-2}^{0}+\frac{1}{2}\left[x^2\right]_{0}^{4}=-\frac{1}{2}(0-4)+\frac{1}{2}(16-0)=2+8=\boxed{10}$$

31.32 3)

$$\int_{-1}^{1}|x|dx=\int_{-1}^{0}|x|dx+\int_{0}^{1}|x|dx=\int_{-1}^{0}-xdx+\int_{0}^{1}xdx=\left[-\frac{x^2}{2}\right]_{-1}^{0}+\left[\frac{x^2}{2}\right]_{0}^{1}=$$

$$=-\frac{1}{2}\left[x^2\right]_{-1}^{0}+\frac{1}{2}\left[x^2\right]_{0}^{1}=-\frac{1}{2}(0-1)+\frac{1}{2}(1-0)=\frac{1}{2}+\frac{1}{2}=1$$

31.32 4)

$$\int_{-3}^{6}x|x|dx=\int_{-3}^{0}x|x|dx+\int_{0}^{6}x|x|dx=\int_{-3}^{0}x\cdot(-x)dx+\int_{0}^{6}x\cdot xdx=$$

$$=\int_{-3}^{0}-x^2dx+\int_{0}^{6}x^2dx=\left[-\frac{x^3}{3}\right]_{-3}^{0}+\left[\frac{x^3}{3}\right]_{0}^{6}=-\frac{1}{3}\left[x^3\right]_{-3}^{0}+\frac{1}{3}\left[x^3\right]_{0}^{6}=$$

$$=-\frac{1}{3}\left(0^3-(-3)^3\right)+\frac{1}{3}\left(6^3-0^3\right)=-\frac{27}{3}+\frac{216}{3}=-9+72=63$$

31.32 5)

$$\int_0^4 |x-2|\,dx \overset{\substack{x-2>0\Leftrightarrow x>2\\ x-2<0\Leftrightarrow x<2}}{=} \int_0^2 |x-2|\,dx+\int_2^4 |x-2|\,dx=\int_0^2 -x+2dx+\int_2^4 x-2dx=$$

$$=\left[-\frac{x^2}{2}+2x\right]_0^2+\left[\frac{x^2}{2}-2x\right]_2^4=$$

$$=\left(-\frac{2^2}{2}+2\cdot 2\right)-\left(-\frac{0^2}{2}+2\cdot 0\right)+\left(\frac{4^2}{2}-2\cdot 4\right)-\left(\frac{4^2}{2}-2\cdot 4\right)=$$

$$=2-0+0-(-2)=4$$

31.32 6)

$$\int_1^{10} |2x-6|\,dx \overset{\substack{2x-6>0\Leftrightarrow x>3\\ 2x-6<0\Leftrightarrow x<3}}{=} \int_1^3 |2x-6|\,dx+\int_3^{10} |2x-6|\,dx=\int_1^3 -2x+6dx+\int_3^{10} 2x-6dx=$$

$$=\left[-x^2+6x\right]_1^3+\left[x^2-6x\right]_3^{10}=-3^2+6\cdot 3-\left(-1^2+6\cdot 1\right)+10^2-6\cdot 10-\left(3^2-6\cdot 3\right)=$$

$$=-\cancel{9}+18-5+100-60+\cancel{9}=53$$

31.32 7)

$$\int_{-1}^4 |x^2-x|\,dx \overset{\substack{x^2-x>0\Leftrightarrow x<0 \text{ ή } x>1\\ x^2-x<0\Leftrightarrow 0<x<1}}{=} \int_{-1}^0 |x^2-x|\,dx+\int_0^1 |x^2-x|\,dx+\int_1^4 |x^2-x|\,dx=$$

$$=\int_{-1}^0 x^2-xdx+\int_0^1 -x^2+xdx+\int_1^4 x^2-xdx=$$

$$=\left[\frac{x^3}{3}-\frac{x^2}{2}\right]_{-1}^0+\left[-\frac{x^3}{3}+\frac{x^2}{2}\right]_0^1+\left[\frac{x^3}{3}-\frac{x^2}{2}\right]_1^4=$$

$$=\left(\frac{0^3}{3}-\frac{0^2}{2}\right)-\left(\frac{(-1)^3}{3}-\frac{(-1)^2}{2}\right)+\left(-\frac{1^3}{3}+\frac{1^2}{2}\right)-\left(-\frac{0^3}{3}+\frac{0^2}{2}\right)+\left(\frac{4^3}{3}-\frac{4^2}{2}\right)-\left(\frac{1^3}{3}-\frac{1^2}{2}\right)=$$

$$=\cancel{\frac{1}{3}}+\frac{1}{2}-\frac{1}{3}+\frac{1}{2}+\frac{64}{3}-\frac{16}{2}-\cancel{\frac{1}{3}}+\frac{1}{2}=21-\frac{13}{2}=\frac{29}{2}$$

31.32 8)

Είναι

$$x^2-4x+3>0 \overset{\Delta=4,\ x_{1,2}=\frac{4\pm 2}{2}= \begin{cases} \nearrow x_1=\frac{4-2}{2}\Rightarrow x_1=1\\ \searrow x_2=\frac{4+2}{2}\Rightarrow x_2=3\end{cases}}{\Leftrightarrow} x<1 \text{ ή } x>3$$

και

$$x^2-4x+3<0 \Leftrightarrow 1<x<3$$

Επομένως

$$\int_{-1}^6 |x^2-4x+3|\,dx=\int_{-1}^1 |x^2-4x+3|\,dx+\int_1^3 |x^2-4x+3|\,dx+\int_3^6 |x^2-4x+3|\,dx=$$

$$=\int_{-1}^1 x^2-4x+3dx+\int_1^3 -x^2+4x-3dx+\int_3^6 x^2-4x+3dx=$$

$$=\left[\frac{x^3}{3}-2x^2+3x\right]_{-1}^{1}+\left[-\frac{x^3}{3}+2x^2-3x\right]_{1}^{3}+\left[\frac{x^3}{3}-2x^2+3x\right]_{3}^{6}=$$

$$=\left(\frac{1^3}{3}-2\cdot1^2+3\cdot1\right)-\left(\frac{(-1)^3}{3}-2(-1)^2+3(-1)\right)+\left(-\frac{3^3}{3}+2\cdot3^2-3\cdot3\right)-$$

$$-\left(-\frac{1^3}{3}+2\cdot1^2-3\cdot1\right)+\left(\frac{6^3}{3}-2\cdot6^2+3\cdot6\right)-\left(\frac{3^3}{3}-2\cdot3^2+3\cdot3\right)=$$

$$=\frac{1}{3}+1-\left(-\frac{1}{3}-5\right)+\left(\cancel{-9+18-9}\right)-\left(-\frac{1}{3}-1\right)+\left(\cancel{72}-\cancel{72}+18\right)-\left(\cancel{9-18+9}\right)=$$

$$=\frac{1}{3}+1+\frac{1}{3}+5+\frac{1}{3}+1+18=26$$

31.32 9)

$$\int_{-1}^{2}\left(|x-1|-3|x|\right)dx=\int_{-1}^{2}|x-1|dx-\int_{-1}^{2}3|x|dx=$$

$$\overset{\substack{x-1>0\Leftrightarrow x>1\\ x-1<0\Leftrightarrow x<1}}{=}\int_{-1}^{1}|x-1|dx+\int_{1}^{2}|x-1|dx-\int_{-1}^{0}|3x|dx-\int_{0}^{2}|3x|dx=$$

$$=\int_{-1}^{1}(-x+1)dx+\int_{1}^{2}x-1dx-\int_{-1}^{0}(-3x)dx-\int_{0}^{2}3xdx=$$

$$=\left[-\frac{x^2}{2}+x\right]_{-1}^{1}+\left[\frac{x^2}{2}-x\right]_{1}^{2}-\left[-\frac{3x^2}{2}\right]_{-1}^{0}-\left[\frac{3x^2}{2}\right]_{0}^{2}=$$

$$=\left(-\frac{1^2}{2}+1\right)-\left(-\frac{(-1)^2}{2}-1\right)+\left(\frac{2^2}{\cancel{2}}-2\right)-\left(\frac{1^2}{2}-1\right)-\left[0-\left(-\frac{3(-1)^2}{2}\right)\right]-\left(\frac{3\cdot2^2}{2}-0\right)=$$

$$=\left(-\frac{1}{2}+1\right)-\left(-\frac{1}{2}-1\right)-\left(\frac{1}{2}-1\right)-\frac{3}{2}-6=-\cancel{\frac{1}{2}}+1+\cancel{\frac{1}{2}}+1-\frac{1}{2}+1-\frac{3}{2}-6=-5$$

31.32 10)

$$\int_{0}^{5}\left(|x-2|+|x-4|\right)dx=\int_{0}^{5}|x-2|dx+\int_{0}^{5}|x-4|dx=$$

$$\overset{\substack{x-2>0\Leftrightarrow x>2\\ x-2<0\Leftrightarrow x<2\\ x-4>0\Leftrightarrow x>4\\ x-4<0\Leftrightarrow x<4}}{=}\int_{0}^{2}|x-2|dx+\int_{2}^{5}|x-2|dx+\int_{0}^{4}|x-4|dx+\int_{4}^{5}|x-4|dx=$$

$$=\int_{0}^{2}-x+2dx+\int_{2}^{5}x-2dx+\int_{0}^{4}-x+4dx+\int_{4}^{5}x-4dx=$$

$$=\left[-\frac{x^2}{2}+2x\right]_{0}^{2}+\left[\frac{x^2}{2}-2x\right]_{2}^{5}+\left[-\frac{x^2}{2}+4x\right]_{0}^{4}+\left[\frac{x^2}{2}-4x\right]_{4}^{5}=$$

$$=-\frac{2^2}{2}+2\cdot2-0+\left(\frac{5^2}{2}-2\cdot5\right)-\left(\frac{2^2}{2}-2\cdot2\right)-\frac{4^2}{2}+4\cdot4-0+\left(\frac{5^2}{2}-4\cdot5\right)-\left(\frac{4^2}{2}-4\cdot4\right)=$$

$$=\cancel{-2}+4+\frac{25}{2}-10\cancel{-(-2)}\cancel{-8}+16+\frac{25}{2}-20\cancel{-(-8)}=15$$

31.32 11)

Το πολυώνυμο x^3-3x+2 έχει ρίζα το $x_o=1$ οπότε το παραγοντοποιούμε με Horner και έχουμε

$$x^3-3x+2=(x-1)(x^2+x-2)\overset{x^2+x-2:\Delta=9\,,\ x_{1,2}=\frac{-1\pm3}{2}=\begin{cases}\nearrow x_1=\frac{-1-3}{2}\Rightarrow x_1=-2\\ \searrow x_2=\frac{-1+3}{2}\Rightarrow x_2=1\end{cases}}{=}(x-1)(x+2)(x-1)=$$
$$=(x-1)^2(x+2)$$

Επομένως είναι

$x^3-3x+2>0\Leftrightarrow x>-2$

$x^3-3x+2<0\Leftrightarrow x<-2$

Έτσι τώρα έχουμε

$$\int_{-3}^{2}\left|x^3-3x+2\right|dx=\int_{-3}^{-2}\left|x^3-3x+2\right|dx+\int_{-2}^{2}\left|x^3-3x+2\right|dx=$$

$$=\int_{-3}^{-2}-x^3+3x-2dx+\int_{-2}^{2}x^3-3x+2dx=$$

$$=\left[-\frac{x^4}{4}+\frac{3x^2}{2}-2x\right]_{-3}^{-2}+\left[\frac{x^4}{4}-\frac{3x^2}{2}+2x\right]_{-2}^{2}=$$

$$=\left(-\frac{(-2)^4}{4}+\frac{3(-2)^2}{2}-2(-2)\right)-\left(-\frac{(-3)^4}{4}+\frac{3(-3)^2}{2}-2(-3)\right)+$$

$$+\left(\frac{2^4}{4}-\frac{3\cdot2^2}{2}+2\cdot2\right)-\left(\frac{(-2)^4}{4}-\frac{3(-2)^2}{2}+2(-2)\right)=$$

$$=\left(\cancel{-4}+6+\cancel{4}\right)-\left(-\frac{81}{4}+\frac{27}{2}+6\right)+(4-6+4)-\left(\cancel{4}-6-\cancel{4}\right)=$$

$$=\cancel{6}+\frac{81}{4}-\frac{27}{2}-\cancel{6}+2+6=\frac{59}{4}$$

31.32 12)

Γνωρίζουμε ότι

αν $x\in\left(0,\frac{\pi}{4}\right)\Rightarrow ημx<συνx$

αν $x\in\left(\frac{\pi}{4},\frac{\pi}{2}\right)\Rightarrow ημx>συνx$

Επομένως

$$\int_0^{\frac{\pi}{2}} \left|\eta\mu x - \sigma\upsilon\nu x\right| dx = \int_0^{\frac{\pi}{4}} \left|\eta\mu x - \sigma\upsilon\nu x\right| dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left|\eta\mu x - \sigma\upsilon\nu x\right| dx =$$

$$= \int_0^{\frac{\pi}{4}} -\eta\mu x + \sigma\upsilon\nu x dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \eta\mu x - \sigma\upsilon\nu x dx =$$

$$= \left[\sigma\upsilon\nu x + \eta\mu x\right]_0^{\frac{\pi}{4}} + \left[-\sigma\upsilon\nu x - \eta\mu x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} =$$

$$= \left(\sigma\upsilon\nu\frac{\pi}{4} + \eta\mu\frac{\pi}{4}\right) - \left(\sigma\upsilon\nu 0 + \eta\mu 0\right) + \left(-\sigma\upsilon\nu\frac{\pi}{2} - \eta\mu\frac{\pi}{2}\right) - \left(-\sigma\upsilon\nu\frac{\pi}{4} - \eta\mu\frac{\pi}{4}\right) =$$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - 1 - 1 + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \frac{4\sqrt{2}}{2} - 2 = 2\sqrt{2} - 2$$