

$$1) \left. \begin{array}{l} \int_a^b f(x) dx = 0 \\ f(x) \geq 0 \text{ για κάθε } x \in [\alpha, \beta] \end{array} \right\} \Rightarrow f(x) = 0, \text{ για κάθε } x \in [\alpha, \beta]$$

$$2) \int_a^b f^2(x) dx = 0 \Rightarrow f(x) = 0, \text{ για κάθε } x \in [\alpha, \beta]$$

$$3) \int_a^b [f(x) - 4x]^2 dx = 0 \Rightarrow f(x) - 4x = 0 \Rightarrow f(x) = 4x, \text{ για κάθε } x \in [\alpha, \beta]$$

$$4) \int_a^b [f(x) - 2x + 1]^4 dx = 0 \Rightarrow f(x) - 2x + 1 = 0 \Rightarrow f(x) = 2x - 1, \text{ για κάθε } x \in [\alpha, \beta]$$

$$5) \int_a^b [f^2(x) - 2xf(x) + x^2] dx = 0 \Rightarrow$$

$$\Rightarrow \int_a^b [f(x) - x]^2 dx = 0 \Rightarrow f(x) - x = 0 \Rightarrow f(x) = x, \text{ για κάθε } x \in [\alpha, \beta]$$