Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Β

31.27 1)

Έστω $I=\int_0^{\frac{\pi}{4}} e^x ημx dx$. Τότε

$$I=\int_0^{\frac{\pi}{4}} e^x ημx dx=\int_0^{\frac{\pi}{4}} \left(e^x\right)' ημx dx=\left[e^x ημx\right]_0^{\frac{\pi}{4}}-\int_0^{\frac{\pi}{4}} e^x \left(ημx\right)' dx=$$

$$=e^{\frac{\pi}{4}} ημ\frac{\pi}{4}-e^0 ημ0-\int_0^{\frac{\pi}{4}} e^x συνx dx=\frac{e^{\frac{\pi}{4}}\sqrt{2}}{2}-\int_0^{\frac{\pi}{4}} e^x συνx dx=$$

$$=\frac{e^{\frac{\pi}{4}}\sqrt{2}}{2}-\int_0^{\frac{\pi}{4}} \left(e^x\right)' συνx dx=\frac{e^{\frac{\pi}{4}}\sqrt{2}}{2}-\left(\left[e^x συνx\right]_0^{\frac{\pi}{4}}-\int_0^{\frac{\pi}{4}} e^x \left(συνx\right)' dx\right)=$$

$$=\frac{e^{\frac{\pi}{4}}\sqrt{2}}{2}-\left(e^{\frac{\pi}{4}} συν\frac{\pi}{4}-e^0 συν0-\int_0^{\frac{\pi}{4}} e^x \left(-ημx\right) dx\right)=$$

$$=\cancel{\frac{e^{\frac{\pi}{4}}\sqrt{2}}{2}}-\cancel{\frac{e^{\frac{\pi}{4}}\sqrt{2}}{2}}+1-\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} e^x συνx dx=$$

$$=1-\int_0^{\frac{\pi}{4}} e^x ημx dx=1-I$$

Επομένως

$$I=1-I\Rightarrow 2I=1\Rightarrow I=\frac{1}{2}$$

31.27 2)

Έστω $I=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} e^x συνx\, dx$. Τότε

$$I=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} e^x συνx\, dx=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(e^x\right)' συνx dx=\left[e^x συνx\right]_{-\frac{\pi}{4}}^{\frac{\pi}{4}}-\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} e^x \left(συνx\right)' dx=$$

$$=e^{\frac{\pi}{4}} συν\frac{\pi}{4}-e^{-\frac{\pi}{4}} συν\left(-\frac{\pi}{4}\right)-\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} e^x \left(-ημx\right) dx=\frac{e^{\frac{\pi}{4}}\sqrt{2}}{2}-\frac{e^{-\frac{\pi}{4}}\sqrt{2}}{2}+\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} e^x ημx dx=$$

$$=\frac{e^{\frac{\pi}{4}}\sqrt{2}}{2}-\frac{e^{-\frac{\pi}{4}}\sqrt{2}}{2}+\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(e^x\right)' ημx dx=\frac{e^{\frac{\pi}{4}}\sqrt{2}}{2}-\frac{e^{-\frac{\pi}{4}}\sqrt{2}}{2}+\left[e^x ημx\right]_{-\frac{\pi}{4}}^{\frac{\pi}{4}}-\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} e^x \left(ημx\right)' dx$$

$$=\frac{e^{\frac{\pi}{4}}\sqrt{2}}{2}-\frac{e^{-\frac{\pi}{4}}\sqrt{2}}{2}+e^{\frac{\pi}{4}} ημ\frac{\pi}{4}-e^{-\frac{\pi}{4}} ημ\left(-\frac{\pi}{4}\right)-\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} e^x συνx dx=$$

$$=\frac{e^{\frac{\pi}{4}}\sqrt{2}}{2}-\cancel{\frac{e^{-\frac{\pi}{4}}\sqrt{2}}{2}}+\frac{e^{\frac{\pi}{4}}\sqrt{2}}{2}+\cancel{\frac{e^{-\frac{\pi}{4}}\sqrt{2}}{2}}-\int_0^{\frac{\pi}{4}} e^x ημx dx=$$

$$=\frac{\cancel{2}e^{\frac{\pi}{4}}\sqrt{2}}{\cancel{2}}-\int_0^{\frac{\pi}{4}}e^x ημxdx=e^{\frac{\pi}{4}}\sqrt{2}-\int_0^{\frac{\pi}{4}}e^x ημxdx=e^{\frac{\pi}{4}}\sqrt{2}-I$$

Επομένως

$$I=e^{\frac{\pi}{4}}\sqrt{2}-I\Rightarrow 2I=e^{\frac{\pi}{4}}\sqrt{2}\Rightarrow I=\frac{e^{\frac{\pi}{4}}\sqrt{2}}{2}$$

31.27 3)

Έστω $I=\int_{-\pi}^{2\pi}e^x ημx\,dx$. Τότε

$$I=\int_{-\pi}^{2\pi}e^x ημx\,dx=\int_{-\pi}^{2\pi}\left(e^x\right)' ημxdx=\left[e^x ημx\right]_{-\pi}^{2\pi}-\int_{-\pi}^{2\pi}e^x\left(ημx\right)'dx=$$

$$=e^{2\pi}ημ2\pi-e^{-\pi}ημ\left(-\pi\right)-\int_{-\pi}^{2\pi}e^x συνxdx=-\int_{-\pi}^{2\pi}e^x συνxdx=$$

$$=-\int_{-\pi}^{2\pi}\left(e^x\right)' συνxdx=-\left(\left[e^x συνx\right]_{-\pi}^{2\pi}-\int_{-\pi}^{2\pi}e^x\left(συνx\right)'dx\right)=$$

$$=-\left(e^{2\pi}συν2\pi-e^{-\pi}συν\left(-\pi\right)-\int_{-\pi}^{2\pi}e^x\left(-ημx\right)dx\right)=$$

$$=-\left(e^{2\pi}+e^{-\pi}+\int_{-\pi}^{2\pi}e^x ημxdx\right)=-e^{2\pi}-e^{-\pi}-\int_{-\pi}^{2\pi}e^x ημxdx=-e^{2\pi}-e^{-\pi}-I$$

Επομένως

$$I=-e^{2\pi}-e^{-\pi}-I\Rightarrow 2I=-e^{2\pi}-e^{-\pi}\Rightarrow I=\frac{-e^{2\pi}-e^{-\pi}}{2}$$

31.27 4)

Έστω $I=\int_{-\frac{\pi}{4}}^{0}e^x συνxdx$. Τότε

$$I=\int_{-\frac{\pi}{4}}^{0}e^x συνx\,dx=\int_{-\frac{\pi}{4}}^{0}\left(e^x\right)' συνxdx=\left[e^x συνx\right]_{-\frac{\pi}{4}}^{0}-\int_{-\frac{\pi}{4}}^{0}e^x\left(συνx\right)'dx=$$

$$=e^0συν0-e^{-\frac{\pi}{4}}συν\left(-\frac{\pi}{4}\right)-\int_{-\frac{\pi}{4}}^{0}e^x\left(-ημx\right)dx=1-\frac{e^{-\frac{\pi}{4}}\sqrt{2}}{2}+\int_{-\frac{\pi}{4}}^{0}e^x ημxdx=$$

$$=1-\frac{e^{-\frac{\pi}{4}}\sqrt{2}}{2}+\int_{-\frac{\pi}{4}}^{0}\left(e^x\right)' ημxdx=1-\frac{e^{-\frac{\pi}{4}}\sqrt{2}}{2}+\left[e^x ημx\right]_{-\frac{\pi}{4}}^{0}-\int_{-\frac{\pi}{4}}^{0}e^x\left(ημx\right)'dx$$

$$=1-\frac{e^{-\frac{\pi}{4}}\sqrt{2}}{2}+e^0ημ0-e^{-\frac{\pi}{4}}ημ\left(-\frac{\pi}{4}\right)-\int_{-\frac{\pi}{4}}^{0}e^x συνxdx=$$

$$=1-\cancel{\frac{e^{-\frac{\pi}{4}}\sqrt{2}}{2}}+\cancel{\frac{e^{-\frac{\pi}{4}}\sqrt{2}}{2}}-\int_0^{\frac{\pi}{4}}e^x ημxdx=1-I$$

Επομένως

$$I=1-I\Rightarrow 2I=1\Rightarrow I=\frac{1}{2}$$

31.27 5)

Έστω $I=\int_0^{\pi}e^x\sigma\upsilon\nu 2x dx$. Τότε

$$I=\int_0^{\pi}e^x\sigma\upsilon\nu 2xdx=\int_0^{\pi}\left(e^x\right)'\sigma\upsilon\nu 2xdx=\left[e^x\sigma\upsilon\nu 2x\right]_0^{\pi}-\int_0^{\pi}e^x\left(\sigma\upsilon\nu 2x\right)'dx=$$

$$=e^{\pi}\sigma\upsilon\nu 2\pi-e^0\sigma\upsilon\nu 0-\int_0^{\pi}e^x\left(-2\eta\mu 2x\right)dx=e^{\pi}-1+2\int_0^{\pi}e^x\eta\mu 2xdx=$$

$$=e^{\pi}-1+2\int_0^{\pi}\left(e^x\right)'\eta\mu 2xdx=e^{\pi}-1+2\left[e^x\eta\mu 2x\right]_0^{\pi}-2\int_0^{\pi}e^x\left(\eta\mu 2x\right)'dx=$$

$$=e^{\pi}-1+2\left(e^{\pi}\eta\mu 2\pi-e^0\eta\mu 0\right)-2\int_0^{\pi}e^x 2\sigma\upsilon\nu 2xdx=$$

$$=e^{\pi}-1-4\int_0^{\pi}e^x\sigma\upsilon\nu 2xdx=e^{\pi}-1-4I$$

Επομένως

$$I=e^{\pi}-1-4I\Rightarrow 5I=e^{\pi}-1\Rightarrow I=\frac{e^{\pi}-1}{5}$$

31.27 6)

Έστω $I=\int_0^{2\pi}e^x\eta\mu 2x\,dx$. Τότε

$$I=\int_0^{2\pi}e^x\eta\mu 2x\,dx=\int_0^{2\pi}\left(e^x\right)'\eta\mu 2xdx=\left[e^x\eta\mu 2x\right]_0^{2\pi}-\int_0^{2\pi}e^x\left(\eta\mu 2x\right)'dx=$$

$$=e^{2\pi}\eta\mu 4\pi-e^0\eta\mu 0-\int_0^{2\pi}2e^x\sigma\upsilon\nu 2xdx=-2\int_0^{2\pi}e^x\sigma\upsilon\nu 2xdx=$$

$$=-2\int_0^{2\pi}\left(e^x\right)'\sigma\upsilon\nu 2xdx=-2\left(\left[e^x\sigma\upsilon\nu 2x\right]_0^{2\pi}-\int_0^{2\pi}e^x\left(\sigma\upsilon\nu 2x\right)'dx\right)=$$

$$=-2\left(e^{2\pi}\sigma\upsilon\nu 2\pi-e^0\sigma\upsilon\nu 0-\int_0^{2\pi}e^x\left(-2\eta\mu 2x\right)dx\right)=$$

$$=-2\left(e^{2\pi}-1+2\int_0^{2\pi}e^x\eta\mu 2xdx\right)=-2e^{2\pi}+2-4\int_0^{2\pi}e^x\eta\mu 2xdx=-2e^{2\pi}+2-4I$$

Επομένως

$$I=-2e^{2\pi}+2-4I\Rightarrow 5I=-2e^{2\pi}+2\Rightarrow I=\frac{-2e^{2\pi}+2}{5}$$

31.27 7)

Έστω $I=\int_{\pi}^{3\pi}e^{2x}\sigma\upsilon\nu x\,dx$. Τότε

$$I=\int_{\pi}^{3\pi}e^{2x}\sigma\upsilon\nu x\,dx=\int_{\pi}^{3\pi}\left(\frac{e^{2x}}{2}\right)'\sigma\upsilon\nu xdx=\left[\frac{e^{2x}\sigma\upsilon\nu x}{2}\right]_{\pi}^{3\pi}-\int_{\pi}^{3\pi}\frac{e^{2x}}{2}\left(\sigma\upsilon\nu x\right)'dx=$$

$$=\frac{e^{6\pi}\sigma\upsilon\nu 3\pi}{2}-\frac{e^{2\pi}\sigma\upsilon\nu\pi}{2}-\int_{\pi}^{3\pi}\frac{e^{2x}}{2}\left(-\eta\mu x\right)dx=\frac{-e^{6\pi}+e^{2\pi}}{2}+\frac{1}{2}\int_{\pi}^{3\pi}e^{2x}\eta\mu xdx=$$

$$= \frac{-e^{6\pi}+e^{2\pi}}{2} + \frac{1}{2}\int_{\pi}^{3\pi}\left(\frac{e^{2x}}{2}\right)' \eta\mu x dx =$$

$$= \frac{-e^{6\pi}+e^{2\pi}}{2} + \frac{1}{2}\left(\left[\frac{e^{2x}\eta\mu x}{2}\right]_{\pi}^{3\pi} - \int_{\pi}^{3\pi}\frac{e^{2x}}{2}(\eta\mu x)' dx\right) =$$

$$= \frac{-e^{6\pi}+e^{2\pi}}{2} + \frac{1}{2}\left(\frac{e^{6\pi}\eta\mu 3\pi}{2} - \frac{e^{2\pi}\eta\mu\pi}{2} - \frac{1}{2}\int_{\pi}^{3\pi} e^{2x}\sigma\upsilon\nu x dx\right) =$$

$$= \frac{-e^{6\pi}+e^{2\pi}}{2} - \frac{1}{4}\int_{\pi}^{3\pi} e^{2x}\sigma\upsilon\nu x dx = \frac{-e^{6\pi}+e^{2\pi}}{2} - \frac{1}{4} I$$

Επομένως

$$I = \frac{-e^{6\pi}+e^{2\pi}}{2} - \frac{1}{4}I \Rightarrow I + \frac{1}{4}I = \frac{-e^{6\pi}+e^{2\pi}}{2} \Rightarrow \frac{5}{\cancel{4}_2} I = \frac{-e^{6\pi}+e^{2\pi}}{\cancel{2}} \Rightarrow I = 2\frac{-e^{6\pi}+e^{2\pi}}{5}$$

31.27 8)

Έστω $I = \int_0^{\pi} e^{2x}\eta\mu 3x\, dx$. Τότε

$$I = \int_0^{\pi} e^{2x}\eta\mu 3x\, dx = \int_0^{\pi}\left(\frac{e^{2x}}{2}\right)' \eta\mu 3x dx = \left[\frac{e^{2x}\eta\mu 3x}{2}\right]_0^{\pi} - \int_0^{\pi}\frac{e^{2x}}{2}(\eta\mu 3x)' dx =$$

$$= \frac{e^{2\pi}\eta\mu 3\pi}{2} - \frac{e^{0}\eta\mu 0}{2} - \int_0^{\pi}\frac{e^{2x}}{2} 3\sigma\upsilon\nu 3x dx = -\frac{3}{2}\int_0^{\pi} e^{2x}\sigma\upsilon\nu 3x dx =$$

$$= -\frac{3}{2}\int_0^{\pi}\left(\frac{e^{2x}}{2}\right)' \sigma\upsilon\nu 3x dx = -\frac{3}{2}\left(\left[\frac{e^{2x}\sigma\upsilon\nu 3x}{2}\right]_0^{\pi} - \int_0^{\pi}\frac{e^{2x}}{2}(\sigma\upsilon\nu 3x)' dx\right) =$$

$$= -\frac{3}{2}\left(\frac{e^{2\pi}\sigma\upsilon\nu 3\pi}{2} - \frac{e^{0}\sigma\upsilon\nu 0}{2} - \int_0^{\pi}\frac{e^{2x}}{2}(-3\eta\mu 3x) dx\right) =$$

$$= -\frac{3}{2}\left(-\frac{e^{2\pi}}{2} - \frac{1}{2} + \frac{3}{2}\int_0^{\pi}\frac{e^{2x}\eta\mu 3x}{2} dx\right) = \frac{3e^{2\pi}}{4} + \frac{3}{4} - \frac{9}{4}\int_0^{\pi}\frac{e^{2x}\eta\mu 3x}{2} dx = \frac{3e^{2\pi}}{4} + \frac{3}{4} - \frac{9}{4} I$$

Επομένως

$$I = \frac{3e^{2\pi}}{4} + \frac{3}{4} - \frac{9}{4}I \Rightarrow I + \frac{9}{4}I = \frac{3e^{2\pi}+3}{4} \Rightarrow \frac{13}{\cancel{4}} I = \frac{3(e^{2\pi}+1)}{\cancel{4}} \Rightarrow I = \frac{3(e^{2\pi}+1)}{13}$$

31.27 9)

Έστω $I = \int_0^{\frac{\pi}{4}} e^{-x}\eta\mu 2x dx$. Τότε

$$I = \int_0^{\frac{\pi}{4}} e^{-x}\eta\mu 2x dx = \int_0^{\frac{\pi}{4}} (-e^{-x})' \eta\mu 2x dx = \left[-e^{-x}\eta\mu 2x\right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} -e^{-x}(\eta\mu 2x)' dx =$$

$$= -e^{-\frac{\pi}{4}}\eta\mu \cancel{2}\frac{\pi}{\cancel{4}_2} + e^{0}\eta\mu 0 + \int_0^{\frac{\pi}{4}} e^{-x} 2\sigma\upsilon\nu 2x dx = -e^{-\frac{\pi}{4}} + 2\int_0^{\frac{\pi}{4}} e^{-x}\sigma\upsilon\nu 2x dx =$$

$$=-e^{-\frac{\pi}{4}}+2\int_0^{\frac{\pi}{4}}\left(-e^{-x}\right)'\sigma\upsilon\nu 2x dx=-e^{-\frac{\pi}{4}}+2\left(\left[-e^{-x}\sigma\upsilon\nu 2x\right]_0^{\frac{\pi}{4}}-\int_0^{\frac{\pi}{4}}-e^{-x}\left(\sigma\upsilon\nu 2x\right)'dx\right)=$$

$$=-e^{-\frac{\pi}{4}}+2\left(-e^{-\frac{\pi}{4}}\sigma\upsilon\nu \cancel{2}\frac{\pi}{\cancel{4}_2}+e^0\sigma\upsilon\nu 0+\int_0^{\frac{\pi}{4}}e^{-x}\left(-2\eta\mu 2x\right)dx\right)=$$

$$=-e^{-\frac{\pi}{4}}+2\left(1-2\int_0^{\frac{\pi}{4}}e^{-x}\eta\mu 2x dx\right)=-e^{-\frac{\pi}{4}}+2-4\int_0^{\frac{\pi}{4}}e^{-x}\eta\mu 2x dx=-e^{-\frac{\pi}{4}}+2-4I$$

Επομένως

$$I=-e^{-\frac{\pi}{4}}+2-4I\Rightarrow 5I=-e^{-\frac{\pi}{4}}+2\Rightarrow I=\frac{2-e^{-\frac{\pi}{4}}}{5}$$

31.27 10)

Έστω $I=\int_{\frac{\pi}{2}}^{\pi}\frac{\eta\mu 2x}{e^x}dx=\int_{\frac{\pi}{2}}^{\pi}e^{-x}\eta\mu 2x dx$. Τότε

$$I=\int_{\frac{\pi}{2}}^{\pi}e^{-x}\eta\mu 2x dx=\int_{\frac{\pi}{2}}^{\pi}\left(-e^{-x}\right)'\eta\mu 2x dx=\left[-e^{-x}\eta\mu 2x\right]_{\frac{\pi}{2}}^{\pi}-\int_{\frac{\pi}{2}}^{\pi}-e^{-x}\left(\eta\mu 2x\right)'dx=$$

$$=-e^{-\pi}\eta\mu 2\pi+e^{-\frac{\pi}{2}}\eta\mu\cancel{2}\frac{\pi}{\cancel{2}}+\int_{\frac{\pi}{2}}^{\pi}e^{-x}2\sigma\upsilon\nu 2x dx=2\int_{\frac{\pi}{2}}^{\pi}e^{-x}\sigma\upsilon\nu 2x dx=$$

$$=2\int_{\frac{\pi}{2}}^{\pi}\left(-e^{-x}\right)'\sigma\upsilon\nu 2x dx=2\left(\left[-e^{-x}\sigma\upsilon\nu 2x\right]_{\frac{\pi}{2}}^{\pi}-\int_{\frac{\pi}{2}}^{\pi}-e^{-x}\left(\sigma\upsilon\nu 2x\right)'dx\right)=$$

$$=2\left(-e^{-\pi}\sigma\upsilon\nu 2\pi+e^{-\frac{\pi}{2}}\sigma\upsilon\nu\cancel{2}\frac{\pi}{\cancel{2}}+\int_{\frac{\pi}{2}}^{\pi}e^{-x}\left(-2\eta\mu 2x\right)dx\right)=$$

$$=2\left(-e^{-\pi}-e^{-\frac{\pi}{2}}-2\int_{\frac{\pi}{2}}^{\pi}e^{-x}\eta\mu 2x dx\right)=-2e^{-\pi}-2e^{-\frac{\pi}{2}}-4\int_{\frac{\pi}{2}}^{\pi}e^{-x}\eta\mu 2x dx=$$

$$=-2e^{-\pi}-2e^{-\frac{\pi}{2}}-4I$$

Επομένως

$$I=-2e^{-\pi}-2e^{-\frac{\pi}{2}}-4I\Rightarrow 5I=-2e^{-\pi}-2e^{-\frac{\pi}{2}}\Rightarrow I=\frac{-2e^{-\pi}-2e^{-\frac{\pi}{2}}}{5}$$

31.27 11)

Έστω $I=\int_0^{\pi}\frac{\sigma\upsilon\nu 2x}{e^{3x}}dx=\int_0^{\pi}e^{-3x}\sigma\upsilon\nu 2x dx$. Τότε

$$I=\int_0^{\pi}e^{-3x}\sigma\upsilon\nu 2x dx=\int_0^{\pi}\left(\frac{e^{-3x}}{-3}\right)'\sigma\upsilon\nu 2x dx=\left[\frac{e^{-3x}\sigma\upsilon\nu 2x}{-3}\right]_0^{\pi}-\int_0^{\pi}\frac{e^{-3x}}{-3}\left(\sigma\upsilon\nu 2x\right)'dx=$$

$$=\frac{e^{-3\pi}\sigma\upsilon\nu 2\pi}{-3}-\frac{e^0\sigma\upsilon\nu 0}{-3}+\frac{1}{3}\int_0^{\pi}e^{-3x}\left(-2\eta\mu 2x\right)dx=\frac{e^{-3\pi}-1}{-3}-\frac{2}{3}\int_0^{\pi}e^{-3x}\eta\mu 2x dx=$$

$$=\frac{1-e^{-3\pi}}{3}-\frac{2}{3}\int_0^{\pi}\left(\frac{e^{-3x}}{-3}\right)'\eta\mu 2x dx=$$

$$=\frac{1-e^{-3\pi}}{3}-\frac{2}{3}\left(\frac{e^{-3\pi}\eta\mu 2\pi}{-3}-\frac{e^{0}\eta\mu 0}{-3}-\int_0^{\pi}\frac{e^{-3x}}{-3}2\sigma\upsilon\nu 2x dx\right)=$$

$$=\frac{1-e^{-3\pi}}{3}-\frac{2}{3}\left(\frac{2}{3}\int_0^{\pi}e^{-3x}\sigma\upsilon\nu 2x dx\right)=\frac{1-e^{-3\pi}}{3}-\frac{4}{9}\int_0^{\pi}e^{-3x}\sigma\upsilon\nu 2x dx=\frac{1-e^{-3\pi}}{3}-\frac{4}{9}I$$

Επομένως

$$I=\frac{1-e^{-3\pi}}{3}-\frac{4}{9}I\Rightarrow I-\frac{4}{9}I=\frac{1-e^{-3\pi}}{3}\Rightarrow\frac{13}{\cancel{9}\,3}I=\frac{1-e^{-3\pi}}{\cancel{3}}\Rightarrow I=\frac{3\left(1-e^{-3\pi}\right)}{13}$$

31.27 12)

Έστω $I=\int_0^1 e^x 3^x dx$. Τότε

$$I=\int_0^1 e^x 3^x dx=\int_0^1\left(e^x\right)'3^x dx=\left[e^x 3^x\right]_0^1-\int_0^1 e^x\left(3^x\right)'dx=e^1 3^1-e^0 3^0-\int_0^1 e^x 3^x\ln 3dx=$$

$$=3e-1-\ln 3\int_0^1 e^x 3^x dx=3e-1-(\ln 3)I$$

Επομένως

$$I=3e-1-(\ln 3)I\Rightarrow I+(\ln 3)I=3e-1\Rightarrow I(1+\ln 3)=3e-1\Rightarrow I=\frac{3e-1}{1+\ln 3}$$

31.27 13)

Έστω $I=\int_0^{\frac{1}{2}} e^{4x}\cdot 3^{2x}dx$. Τότε

$$I=\int_0^{\frac{1}{2}} e^{4x}\cdot 3^{2x}dx=\int_0^{\frac{1}{2}}\left(\frac{e^{4x}}{4}\right)'3^{2x}dx=\left[\frac{e^{4x}3^{2x}}{4}\right]_0^{\frac{1}{2}}-\int_0^{\frac{1}{2}}\frac{e^{4x}}{4}\left(3^{2x}\right)'dx=$$

$$=\frac{3e^2-1}{4}-\frac{2\ln 3}{4}\int_0^{\frac{1}{2}}e^x 3^{2x}dx=\frac{3e^2-1}{4}-\frac{2\ln 3}{4}\cdot I$$

Επομένως

$$I=\frac{3e^2-1}{4}-\frac{2\ln 3}{4}\cdot I\Rightarrow I+\frac{2\ln 3}{4}\cdot I=\frac{3e^2-1}{4}\Rightarrow\frac{2\ln 3+1}{4}\cdot I=\frac{3e^2-1}{4}\Rightarrow$$

$$\Rightarrow I=\frac{3e^2-1}{2\ln 3+1}$$

31.27 14)

Έστω $I=\int_0^{\frac{\pi}{2}} e^x(\eta\mu x+\sigma\upsilon\nu x)dx$. Τότε

$$I=\int_0^{\frac{\pi}{2}} e^x(\eta\mu x+\sigma\upsilon\nu x)dx=\int_0^{\frac{\pi}{2}}\left(e^x\right)'(\eta\mu x+\sigma\upsilon\nu x)dx=$$

$$=\left[e^x(\eta\mu x+\sigma\upsilon\nu x)\right]_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}e^x(\eta\mu x+\sigma\upsilon\nu x)'dx=$$

$$=e^{\frac{\pi}{2}}\left(\eta\mu\frac{\pi}{2}+\sigma\upsilon\nu\frac{\pi}{2}\right)-e^0(\eta\mu 0+\sigma\upsilon\nu 0)-\int_0^{\frac{\pi}{2}}e^x(\sigma\upsilon\nu x-\eta\mu x)dx=$$

$$=e^{\frac{\pi}{2}}-1-\int_0^{\frac{\pi}{2}}\left(e^x\right)'\left(συνx-ημx\right)dx=$$

$$=e^{\frac{\pi}{2}}-1-\left(\left[e^x\left(συνx-ημx\right)\right]_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}e^x\left(συνx-ημx\right)'dx\right)=$$

$$=e^{\frac{\pi}{2}}-1-\left(e^{\frac{\pi}{2}}\left(συν\frac{\pi}{2}-ημ\frac{\pi}{2}\right)-e^0\left(συν0-ημ0\right)-\int_0^{\frac{\pi}{2}}e^x\left(-ημx-συνx\right)dx\right)=$$

$$=e^{\frac{\pi}{2}}-1-\left(-e^{\frac{\pi}{2}}-1+\int_0^{\frac{\pi}{2}}e^x\left(ημx+συνx\right)dx\right)=$$

$$=e^{\frac{\pi}{2}}-\cancel{1}+e^{\frac{\pi}{2}}+\cancel{1}-\int_0^{\frac{\pi}{2}}e^x\left(ημx+συνx\right)dx=2e^{\frac{\pi}{2}}-I$$

Επομένως

$$I=2e^{\frac{\pi}{2}}-I\Rightarrow 2I=2e^{\frac{\pi}{2}}\Rightarrow I=e^{\frac{\pi}{2}}$$

31.27 15)

Έστω $I=\int_0^{\frac{\pi}{2}}e^x\left(συνx-ημx\right)dx$. Τότε

$$I=\int_0^{\frac{\pi}{2}}e^x\left(συνx-ημx\right)dx=\int_0^{\frac{\pi}{2}}\left(e^x\right)'\left(συνx-ημx\right)dx=$$

$$=\left[e^x\left(συνx-ημx\right)\right]_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}e^x\left(συνx-ημx\right)'dx=$$

$$=e^{\frac{\pi}{2}}\left(συν\frac{\pi}{2}-ημ\frac{\pi}{2}\right)-e^0\left(συν0-ημ0\right)-\int_0^{\frac{\pi}{2}}e^x\left(-ημx-συνx\right)dx=$$

$$=-e^{\frac{\pi}{2}}-1+\int_0^{\frac{\pi}{2}}e^x\left(συνx+ημx\right)dx=-e^{\frac{\pi}{2}}-1+\int_0^{\frac{\pi}{2}}\left(e^x\right)'\left(συνx+ημx\right)dx=$$

$$=-e^{\frac{\pi}{2}}-1+\left(\left[e^x\left(συνx+ημx\right)\right]_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}e^x\left(συνx+ημx\right)'dx\right)=$$

$$=-e^{\frac{\pi}{2}}-1+\left(e^{\frac{\pi}{2}}\left(συν\frac{\pi}{2}+ημ\frac{\pi}{2}\right)-e^0\left(συν0+ημ0\right)-\int_0^{\frac{\pi}{2}}e^x\left(-ημx+συνx\right)dx\right)=$$

$$=-e^{\frac{\pi}{2}}-1+\left(e^{\frac{\pi}{2}}-1-\int_0^{\frac{\pi}{2}}e^x\left(συνx-ημx\right)dx\right)=$$

$$=-\cancel{e^{\frac{\pi}{2}}}-1+\cancel{e^{\frac{\pi}{2}}}-1-\int_0^{\frac{\pi}{2}}e^x\left(συνx-ημx\right)dx=-2-I$$

Επομένως

$$I=-2-I\Rightarrow 2I=-2\Rightarrow I=-1$$

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α) Έστω $I=\int_0^{\frac{\pi}{2}}e^xσυνxdx$. Τότε

$$I=\int_0^{\frac{\pi}{2}}e^xσυνxdx=\int_0^{\frac{\pi}{2}}\left(e^x\right)'συνxdx=\left[e^xσυνx\right]_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}e^x\left(συνx\right)'dx=$$

$$= e^{\frac{\pi}{2}}συν\frac{\pi}{2} - e^0συν0 - \int_0^{\frac{\pi}{2}} e^x(-ημx)\,dx = -1 + \int_0^{\frac{\pi}{2}} e^x ημx dx =$$

$$= -1 + \int_0^{\frac{\pi}{2}} (e^x)' ημx dx = -1 + \left[e^x ημx\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} e^x (ημx)' dx$$

$$= -1 + e^{\frac{\pi}{2}}ημ\frac{\pi}{2} - e^0ημ0 - \int_0^{\frac{\pi}{2}} e^x συνx dx =$$

$$= -1 + e^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} e^x ημx dx = -1 + e^{\frac{\pi}{2}} - I$$

Επομένως

$$I = -1 + e^{\frac{\pi}{2}} - I \Rightarrow 2I = -1 + e^{\frac{\pi}{2}} \Rightarrow I = \frac{e^{\frac{\pi}{2}} - 1}{2}$$

β) Έστω $J = \int_0^{\frac{\pi}{2}} xe^x ημx dx$. Τότε

$$J = \int_0^{\frac{\pi}{2}} xe^x ημx dx = \int_0^{\frac{\pi}{2}} (e^x)' xημx dx =$$

$$= \left[xe^x ημx\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} e^x (xημx)' dx =$$

$$= e^{\frac{\pi}{2}}\frac{\pi}{2}ημ\frac{\pi}{2} - 0 - \int_0^{\frac{\pi}{2}} e^x (ημx + xσυνx) dx =$$

$$= \frac{\pi e^{\frac{\pi}{2}}}{2} - \int_0^{\frac{\pi}{2}} (e^x)' (ημx + xσυνx) dx =$$

$$= \frac{\pi e^{\frac{\pi}{2}}}{2} - \left(\left[e^x(ημx + xσυνx)\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} e^x (ημx + xσυνx)' dx\right) =$$

$$= \frac{\pi e^{\frac{\pi}{2}}}{2} - \left(e^{\frac{\pi}{2}}\left(ημ\frac{\pi}{2} + \frac{\pi}{2}συν\frac{\pi}{2}\right) - 0 - \int_0^{\frac{\pi}{2}} e^x (συνx + συνx - xημx) dx\right) =$$

$$= \frac{\pi e^{\frac{\pi}{2}}}{2} - \left(e^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} e^x (2συνx - xημx) dx\right) = \frac{\pi e^{\frac{\pi}{2}}}{2} - e^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} e^x (2συνx - xημx) dx =$$

$$= \frac{\pi e^{\frac{\pi}{2}} - 2e^{\frac{\pi}{2}}}{2} + \int_0^{\frac{\pi}{2}} e^x 2συνx dx - \int_0^{\frac{\pi}{2}} e^x xημx dx =$$

$$\overset{\text{α) ερώτημα}}{=} \frac{(\pi - 2)e^{\frac{\pi}{2}}}{2} + 2\frac{e^{\frac{\pi}{2}} - 1}{2} - J = \frac{(\pi - 2)e^{\frac{\pi}{2}} + 2e^{\frac{\pi}{2}} - 2}{2} - J = \frac{\pi e^{\frac{\pi}{2}} - 2}{2} - J$$

Επομένως

$$J = \frac{\pi e^{\frac{\pi}{2}} - 2}{2} - J \Rightarrow 2J = \frac{\pi e^{\frac{\pi}{2}} - 2}{2} \Rightarrow J = \frac{\pi e^{\frac{\pi}{2}} - 2}{4}$$