Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Β

31.24 1)

α) $\int_0^{\frac{\pi}{2}} \eta\mu^4 x \cdot \sigma\upsilon\nu^3 x\,dx = \int_0^{\frac{\pi}{2}} \eta\mu^4 x \cdot \sigma\upsilon\nu^2 x\,\sigma\upsilon\nu x dx = \int_0^{\frac{\pi}{2}} \eta\mu^4 x \cdot \left(1-\eta\mu^2 x\right)\sigma\upsilon\nu x dx =$

$$\overset{\substack{\text{θέτουμε } y=\eta\mu x \Rightarrow dy=\sigma\upsilon\nu x dx \\ \text{για } x=0 \Rightarrow y=\eta\mu 0=0 \\ \text{για } x=\frac{\pi}{2} \Rightarrow y=\eta\mu\frac{\pi}{2}=1}}{=} \int_0^1 y^4 \cdot \left(1-y^2\right)dy = \int_0^1 \left(y^4-y^6\right)dy = \left[\frac{y^5}{5}-\frac{y^7}{7}\right]_0^1 = \frac{1}{5}-\frac{1}{7}=\frac{2}{35}$$

β) $\int_0^{\frac{\pi}{2}} \eta\mu^7 x\,dx = \int_0^{\frac{\pi}{2}} \eta\mu^6 x \eta\mu x\,dx = \int_0^{\frac{\pi}{2}} \left(1-\sigma\upsilon\nu^2 x\right)^3 \eta\mu x\,dx =$

$$\overset{\substack{\text{θέτουμε } y=\sigma\upsilon\nu x \Rightarrow dy=-\eta\mu x dx \\ \text{για } x=0 \Rightarrow y=\sigma\upsilon\nu 0=1 \\ \text{για } x=\frac{\pi}{2} \Rightarrow y=\sigma\upsilon\nu\frac{\pi}{2}=0}}{=} -\int_1^0 \left(1-y^2\right)^3 dy = \int_0^1 \left(1-3y^2+3y^4-y^6\right)dy =$$

$$= \left[y-y^3+3\frac{y^5}{5}-\frac{y^7}{7}\right]_0^1 = 1-1^3+3\frac{1^5}{5}-\frac{1^7}{7}=\frac{16}{35}$$

γ) $\int_0^{\frac{\pi}{2}} \frac{\sigma\upsilon\nu^3 x}{1+\eta\mu x}dx = \int_0^{\frac{\pi}{2}} \frac{\sigma\upsilon\nu^2 x}{1+\eta\mu x}\sigma\upsilon\nu x dx = \int_0^{\frac{\pi}{2}} \frac{1-\eta\mu^2 x}{1+\eta\mu x}\sigma\upsilon\nu x dx =$

$$\overset{\substack{\text{θέτουμε } y=\eta\mu x \Rightarrow dy=\sigma\upsilon\nu x dx \\ \text{για } x=0 \Rightarrow y=\eta\mu 0=0 \\ \text{για } x=\frac{\pi}{2} \Rightarrow y=\eta\mu\frac{\pi}{2}=1}}{=} \int_0^1 \frac{1-y^2}{1+y}dy = \int_0^1 \frac{(1-y)\cancel{(1+y)}}{\cancel{1+y}}dy = \left[y-\frac{y^2}{2}\right]_0^1 = 1-\frac{1}{2}=\frac{1}{2}$$

δ) $$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1}{\eta\mu x}dx = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\eta\mu x}{\eta\mu^2 x}dx = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\eta\mu x}{1-\sigma\upsilon\nu^2 x}dx \overset{\substack{\text{θέτουμε } y=\sigma\upsilon\nu x \Rightarrow dy=-\eta\mu x dx \\ \text{για } x=\frac{\pi}{3} \Rightarrow y=\sigma\upsilon\nu\frac{\pi}{3}=\frac{1}{2} \\ \text{για } x=\frac{\pi}{2} \Rightarrow y=\sigma\upsilon\nu\frac{\pi}{2}=0}}{=} -\int_{\frac{1}{2}}^0 \frac{1}{1-y^2}dy =$$

$$= \int_{\frac{1}{2}}^0 \frac{1}{y^2-1}dy$$

Είναι $\frac{1}{y^2-1} = \frac{1}{(y-1)(y+1)}$

Οπότε θα πρέπει να βρούμε τα A και B ώστε $\frac{1}{(y-1)(y+1)} = \frac{A}{y-1}+\frac{B}{y+1}$

Έχουμε

$$\frac{1}{(y-1)(y+1)} = \frac{A}{y-1}+\frac{B}{y+1} \Rightarrow$$

$$\Rightarrow \cancel{(y-1)(y+1)}\frac{1}{\cancel{(y-1)(y+1)}} = \cancel{(y-1)}(y+1)\frac{A}{\cancel{y-1}} + (y-1)\cancel{(y+1)}\frac{B}{\cancel{y+1}} \Rightarrow$$

$$\Rightarrow 1 = A(y+1)+B(y-1) \Rightarrow 1 = Ay+A+By-B \Rightarrow$$

$$\Rightarrow 1 = (A+B)y+A-B \Rightarrow \begin{cases} A+B=0 \\ A-B=1 \end{cases} \overset{\text{λύνουμε το σύστημα}}{\Rightarrow} \begin{cases} A=\frac{1}{2} \\ B=-\frac{1}{2} \end{cases}$$

Οπότε $\frac{1}{(x-1)(x+1)} = \frac{\frac{1}{2}}{x-1}-\frac{\frac{1}{2}}{x+1}$

Άρα το ζητούμενο ολοκλήρωμα είναι

$$=\int_{\frac{1}{2}}^{0}\frac{1}{y^2-1}dy=\int_{\frac{1}{2}}^{0}\frac{\frac{1}{2}}{x-1}-\frac{\frac{1}{2}}{x+1}dx=\frac{1}{2}\int_{\frac{1}{2}}^{0}\frac{1}{x-1}dx-\frac{1}{2}\int_{\frac{1}{2}}^{0}\frac{1}{x+1}dx=$$

$$=\frac{1}{2}\Big[\ln|x-1|\Big]_{\frac{1}{2}}^{0}-\frac{1}{2}\Big[\ln|x+1|\Big]_{\frac{1}{2}}^{0}=\ldots=\frac{\ln 3}{2}$$

31.24 2)

$$\int_0^{\frac{\pi}{2}}\eta\mu^6 x\cdot\sigma\upsilon\nu x\,dx\overset{\substack{\text{θέτουμε } y=\eta\mu x\Rightarrow dy=\sigma\upsilon\nu x dx\\ \text{για } x=0\Rightarrow y=\eta\mu 0=0\\ \text{για } x=\frac{\pi}{2}\Rightarrow y=\eta\mu\frac{\pi}{2}=1}}{=}\int_0^1 y^6\,dy=\left[\frac{y^7}{7}\right]_0^1=\frac{1}{7}$$

31.24 3)

$$\int_0^{\frac{\pi}{2}}\eta\mu^3 x\,\sigma\upsilon\nu^2 x\,dx=\int_0^{\frac{\pi}{2}}\eta\mu^2 x\,\sigma\upsilon\nu^2 x\,\eta\mu x dx=\int_0^{\frac{\pi}{2}}\left(1-\sigma\upsilon\nu^2 x\right)\sigma\upsilon\nu^2 x\,\eta\mu x dx=$$

$$\overset{\substack{\text{θέτουμε } y=\sigma\upsilon\nu x\Rightarrow dy=-\eta\mu x dx\\ \text{για } x=0\Rightarrow y=\sigma\upsilon\nu 0=1\\ \text{για } x=\frac{\pi}{2}\Rightarrow y=\sigma\upsilon\nu\frac{\pi}{2}=0}}{=}\int_1^0-\left(1-y^2\right)y^2\,dy=-\int_1^0 y^2-y^4\,dy=\int_1^0 y^4-y^2\,dy=$$

$$=\left[\frac{y^5}{5}-\frac{y^3}{3}\right]_1^0=0-\left(\frac{1}{5}-\frac{1}{3}\right)=\frac{2}{15}$$

31.24 4)

$$\int_0^{\frac{\pi}{2}}\eta\mu^3 x\sigma\upsilon\nu^4 x dx=\int_0^{\frac{\pi}{2}}\eta\mu^2 x\sigma\upsilon\nu^4 x\eta\mu x dx=\int_0^{\frac{\pi}{2}}\left(1-\sigma\upsilon\nu^2 x\right)\sigma\upsilon\nu^4 x\eta\mu x dx=$$

$$\overset{\substack{\text{θέτουμε } y=\sigma\upsilon\nu x\Rightarrow dy=-\eta\mu x dx\\ \text{για } x=0\Rightarrow y=\sigma\upsilon\nu 0=1\\ \text{για } x=\frac{\pi}{2}\Rightarrow y=\sigma\upsilon\nu\frac{\pi}{2}=0}}{=}-\int_1^0\left(1-y^2\right)y^4 dy=\int_0^1 y^4-y^6 dy=\left[\frac{y^5}{5}-\frac{y^7}{7}\right]_0^1=$$

$$=\left(\frac{1^5}{5}-\frac{1^7}{7}\right)=\frac{2}{35}$$

31.24 5)

$$\int_0^{\frac{\pi}{2}}\eta\mu^2 x\sigma\upsilon\nu^5 x dx=\int_0^{\frac{\pi}{2}}\eta\mu^2 x\sigma\upsilon\nu^4 x\sigma\upsilon\nu x dx=\int_0^{\frac{\pi}{2}}\eta\mu^2 x\left(\sigma\upsilon\nu^2 x\right)^2\sigma\upsilon\nu x dx=$$

$$=\int_0^{\frac{\pi}{2}}\eta\mu^2 x\left(1-\eta\mu^2 x\right)^2\sigma\upsilon\nu x dx\overset{\substack{\text{θέτουμε } y=\eta\mu x\Rightarrow dy=\sigma\upsilon\nu x dx\\ \text{για } x=0\Rightarrow y=\eta\mu 0=0\\ \text{για } x=\frac{\pi}{2}\Rightarrow y=\eta\mu\frac{\pi}{2}=1}}{=}\int_0^1 y^2\left(1-y^2\right)^2 dy=$$

$$=\int_0^1 y^2\left(1-2y^2+y^4\right)dy=\int_0^1 y^2-2y^4+y^6 dy=\left[\frac{y^3}{3}-2\frac{y^5}{5}+\frac{y^7}{7}\right]_0^1=$$

$$=\frac{1^3}{3}-2\frac{1^5}{5}+\frac{1^7}{7}=\frac{8}{105}$$

31.24 6)

$$\int_{-\pi}^{2\pi}\eta\mu^5 x\sigma\upsilon\nu^4 x\,dx=\int_{-\pi}^{2\pi}\eta\mu^4 x\sigma\upsilon\nu^4 x\eta\mu x dx=\int_{-\pi}^{2\pi}\left(1-\sigma\upsilon\nu^2 x\right)^2\sigma\upsilon\nu^4 x\eta\mu x dx=$$

$$\overset{\substack{\text{θέτουμε } y=\text{συν}x \Rightarrow dy=-\text{ημ}x dx \\ \text{για } x=-\pi \Rightarrow y=\text{συν}(-\pi)=-1 \\ \text{για } x=2\pi \Rightarrow y=\text{συν}2\pi=0}}{=} -\int_{-1}^{1}\left(1-y^2\right)^2 y^4 dx = \int_{1}^{-1}\left(1-2y^2+y^4\right)y^4 dx =$$

$$=\int_{1}^{-1}\left(y^4-2y^6+y^8\right)dx=\left[\frac{y^5}{5}-2\frac{y^7}{7}+\frac{y^9}{9}\right]_1^{-1}=$$

$$=\frac{(-1)^5}{5}-2\frac{(-1)^7}{7}+\frac{(-1)^9}{9}-\left(\frac{1^5}{5}-2\frac{1^7}{7}+\frac{1^9}{9}\right)=-\frac{1}{5}+\frac{2}{7}-\frac{1}{9}-\left(\frac{1}{5}-\frac{2}{7}+\frac{1}{9}\right)=-\frac{16}{315}$$

31.24 7)

$$\int_0^{\frac{\pi}{2}} \text{ημ}^2 x\text{συν}^3 x\,dx=\int_0^{\frac{\pi}{2}} \text{ημ}^2 x\text{συν}^2 x\text{συν}x dx=\int_0^{\frac{\pi}{2}} \text{ημ}^2 x\left(1-\text{ημ}^2 x\right)\text{συν}x dx=$$

$$\overset{\substack{\text{θέτουμε } y=\text{ημ}x \Rightarrow dy=\text{συν}x dx \\ \text{για } x=0 \Rightarrow y=\text{ημ}0=0 \\ \text{για } x=\frac{\pi}{2} \Rightarrow y=\text{ημ}\frac{\pi}{2}=1}}{=} \int_0^1 y^2\cdot\left(1-y^2\right)dy=\int_0^1\left(y^2-y^4\right)dy=\left[\frac{y^3}{3}-\frac{y^5}{5}\right]_0^1=\frac{1}{3}-\frac{1}{5}=\frac{2}{15}$$

31.24 8)

$$\int_0^{\frac{\pi}{4}} \text{ημ}^4 x\text{συν}^5 x\,dx=\int_0^{\frac{\pi}{4}} \text{ημ}^4 x\text{συν}^4 x\text{συν}x dx=\int_0^{\frac{\pi}{2}} \text{ημ}^4 x\left(1-\text{ημ}^2 x\right)^2\text{συν}x dx=$$

$$\overset{\substack{\text{θέτουμε } y=\text{ημ}x \Rightarrow dy=\text{συν}x dx \\ \text{για } x=0 \Rightarrow y=\text{ημ}0=0 \\ \text{για } x=\frac{\pi}{4} \Rightarrow y=\text{ημ}\frac{\pi}{2}=1}}{=} \int_0^1 y^4\cdot\left(1-y^2\right)^2 dy=\int_0^1 y^4\cdot\left(1-2y^2+y^4\right)dy=$$

$$=\int_0^1\left(y^4-2y^6+y^8\right)dy=\left[\frac{y^5}{5}-2\frac{y^7}{7}+\frac{y^9}{9}\right]_0^1=\frac{1^3}{5}-2\frac{1^5}{7}+\frac{1^7}{9}=\frac{8}{315}$$

31.24 9)

$$\int_0^{\frac{\pi}{2}} \text{συν}^3 x dx=\int_0^{\frac{\pi}{2}} \text{συν}^2 x\text{συν}x dx=\int_0^{\frac{\pi}{2}}\left(1-\text{ημ}^2 x\right)\text{συν}x dx \overset{\substack{\text{θέτουμε } y=\text{ημ}x \Rightarrow dy=\text{συν}x dx \\ \text{για } x=0 \Rightarrow y=\text{ημ}0=0 \\ \text{για } x=\frac{\pi}{4} \Rightarrow y=\text{ημ}\frac{\pi}{2}=1}}{=}$$

$$\overset{\substack{\text{θέτουμε } y=\text{ημ}x \Rightarrow dy=\text{συν}x dx \\ \text{για } x=0 \Rightarrow y=\text{ημ}0=0 \\ \text{για } x=\frac{\pi}{4} \Rightarrow y=\text{ημ}\frac{\pi}{2}=1}}{=} \int_0^1 1-y^2 dx=\left[y-\frac{y^3}{3}\right]_0^1=1-\frac{1}{3}=\frac{2}{3}$$

31.24 10)

$$\int_0^{\frac{\pi}{2}} \text{ημ}^3 x\,dx=\int_0^{\frac{\pi}{2}} \text{ημ}^2 x\text{ημ}x\,dx=\int_0^{\frac{\pi}{2}}\left(1-\text{συν}^2 x\right)\text{ημ}x\,dx=$$

$$\overset{\substack{\text{θέτουμε } y=\text{συν}x \Rightarrow dy=-\text{ημ}x dx \\ \text{για } x=0 \Rightarrow y=\text{συν}0=1 \\ \text{για } x=\frac{\pi}{2} \Rightarrow y=\text{συν}\frac{\pi}{2}=0}}{=} -\int_1^0\left(1-y^2\right)dy=\int_0^1\left(1-y^2\right)dy=\left[y-\frac{y^3}{3}\right]_0^1=1-\frac{1}{3}=\frac{2}{3}$$

31.24 11)

$$\int_0^{\frac{\pi}{2}} \text{συν}^5 x dx=\int_0^{\frac{\pi}{2}} \text{συν}^4 x\text{συν}x dx=\int_0^{\frac{\pi}{2}}\left(\text{συν}^2 x\right)^2\text{συν}x dx=\int_0^{\frac{\pi}{2}}\left(1-\text{ημ}^2 x\right)^2\text{συν}x dx=$$

θέτουμε y=συνx⇒dy=−ημxdx
για x=0⇒y=συν0=1
για $x=\frac{\pi}{2}\Rightarrow y=\sigma\upsilon\nu\frac{\pi}{2}=0$

$$= \quad -\int_1^0 (1-y^2)^2 dy = \int_0^1 (1-2y^2+y^4)dy = \left[y-2\frac{y^3}{3}+\frac{y^5}{5}\right]_0^1 =$$

$$=1-2\frac{1^3}{3}+\frac{1^5}{5}=\frac{8}{15}$$

31.24 12)

$$\int_0^{\frac{\pi}{3}} \frac{\eta\mu^3 x}{\sigma\upsilon\nu x}dx = \int_0^{\frac{\pi}{3}} \frac{\eta\mu^2 x}{\sigma\upsilon\nu x}\eta\mu x dx = \int_0^{\frac{\pi}{3}} \frac{1-\sigma\upsilon\nu^2 x}{\sigma\upsilon\nu x}\eta\mu x dx =$$

θέτουμε y=συνx⇒dy=−ημxdx
για x=0⇒y=συν0=1
για $x=\frac{\pi}{3}\Rightarrow y=\sigma\upsilon\nu\frac{\pi}{3}=\frac{1}{2}$

$$= \quad -\int_1^{\frac{1}{2}} \frac{1-y^2}{y}dy = \int_{\frac{1}{2}}^1 \frac{1}{y}-\frac{y^{\cancel{2}}}{\cancel{y}}dy = \int_{\frac{1}{2}}^1 \left(\frac{1}{y}-y\right)dy = \left[\ln|y|-\frac{y^2}{2}\right]_{\frac{1}{2}}^1 =$$

$$=\cancel{\ln|1|}-\frac{1^2}{2}-\left(\ln\left|\frac{1}{2}\right|-\frac{\frac{1}{4}}{2}\right)=-\frac{1}{2}-\ln\left|\frac{1}{2}\right|+\frac{1}{8}=-\frac{3}{8}+\ln 2$$

31.24 13)

$$\int_0^{\frac{\pi}{3}} \frac{\eta\mu^5 x}{\sigma\upsilon\nu^4 x}dx = \int_0^{\frac{\pi}{3}} \frac{\eta\mu^4 x}{\sigma\upsilon\nu^4 x}\eta\mu x dx = \int_0^{\frac{\pi}{3}} \frac{(1-\sigma\upsilon\nu^2 x)^2}{\sigma\upsilon\nu^4 x}\eta\mu x dx =$$

θέτουμε y=συνx⇒dy=−ημxdx
για x=0⇒y=συν0=1
για $x=\frac{\pi}{3}\Rightarrow y=\sigma\upsilon\nu\frac{\pi}{3}=\frac{1}{2}$

$$= \quad -\int_1^{\frac{1}{2}} \frac{(1-y^2)^2}{y^4}dy = \int_{\frac{1}{2}}^1 \frac{1-2y^2+y^4}{y^4}dy = \int_{\frac{1}{2}}^1 \frac{1}{y^4}-\frac{2\cancel{y^2}}{y^{\cancel{4}2}}+\frac{y^4}{y^4}dy =$$

$$=\int_{\frac{1}{2}}^1 \left(\frac{1}{y^4}-\frac{2}{y^2}+1\right)dy = \left[\frac{y^{-3}}{-3}-2\frac{y^{-1}}{-1}+y\right]_{\frac{1}{2}}^1 = \left[\frac{1}{-3y^3}+\frac{2}{y}+y\right]_{\frac{1}{2}}^1 =$$

$$=\frac{1}{-3\cdot 1^3}+\frac{2}{1}+1-\left(\frac{1}{-3\left(\frac{1}{2}\right)^3}+\frac{2}{\frac{1}{2}}+\frac{1}{2}\right)=-\frac{1}{3}+3+\frac{8}{3}-4-\frac{1}{2}=\frac{5}{6}$$

31.24 14)

$$\int_0^{\pi} \frac{\eta\mu^3 x}{2+\sigma\upsilon\nu x}dx = \int_0^{\pi} \frac{\eta\mu^2 x}{2+\sigma\upsilon\nu x}\eta\mu x dx = \int_0^{\pi} \frac{1-\sigma\upsilon\nu^2 x}{2+\sigma\upsilon\nu x}\eta\mu x dx =$$

θέτουμε y=συνx⇒dy=−ημxdx
για x=0⇒y=συν0=1
για x=π⇒y=συνπ=−1

$$= \quad -\int_1^{-1} \frac{1-y^2}{2+y}dy = \int_1^{-1} \frac{y^2-1}{y+2}dy = \int_1^{-1} \frac{y^2-4+3}{y+2}dy =$$

$$=\int_1^{-1} \frac{y^2-4}{y+2}dy + \int_1^{-1} \frac{3}{y+2}dy = \int_1^{-1} \frac{(y-2)\cancel{(y+2)}}{\cancel{y+2}}dy + 3\int_1^{-1} \frac{1}{y+2}dy =$$

$$=\int_{1}^{-1}\frac{(y-2)\cancel{(y+2)}}{\cancel{y+2}}dy+3\int_{1}^{-1}\frac{1}{y+2}dy=\left[\frac{y^2}{2}-2y\right]_1^{-1}+3\left[\ln|y+2|\right]_1^{-1}=$$

$$=\left(\frac{(-1)^2}{2}-2(-1)\right)-\left(\frac{1^2}{2}-2\cdot 1\right)+3\left(\cancel{\ln|-1+2|}-\ln|1+2|\right)=$$

$$=\cancel{\frac{1}{2}}+2-\cancel{\frac{1}{2}}+2-3\ln 3=4-3\ln 3$$

31.24 15)

$$\int_0^{\frac{\pi}{6}}\frac{1}{\sigma\upsilon\nu x}dx=\int_0^{\frac{\pi}{6}}\frac{\sigma\upsilon\nu x}{\sigma\upsilon\nu^2 x}dx=\int_0^{\frac{\pi}{6}}\frac{\sigma\upsilon\nu x}{1-\eta\mu^2 x}dx\overset{\substack{\text{θέτουμε } y=\eta\mu x\Rightarrow dy=\sigma\upsilon\nu x dx\\ \text{για } x=0\Rightarrow y=\eta\mu 0=0\\ \text{για } x=\frac{\pi}{6}\Rightarrow y=\eta\mu\frac{\pi}{6}=\frac{1}{2}}}{=}\int_0^{\frac{1}{2}}\frac{1}{1-y^2}dy=$$

$$=\int_{\frac{1}{2}}^{0}\frac{1}{y^2-1}dy$$

Είναι $\frac{1}{y^2-1}=\frac{1}{(y-1)(y+1)}$

Οπότε θα πρέπει να βρούμε τα A και B ώστε $\frac{1}{(y-1)(y+1)}=\frac{A}{y-1}+\frac{B}{y+1}$

Έχουμε

$$\frac{1}{(y-1)(y+1)}=\frac{A}{y-1}+\frac{B}{y+1}\Rightarrow$$

$$\Rightarrow \cancel{(y-1)(y+1)}\frac{1}{\cancel{(y-1)(y+1)}}=\cancel{(y-1)}(y+1)\frac{A}{\cancel{y-1}}+(y-1)\cancel{(y+1)}\frac{B}{\cancel{y+1}}\Rightarrow$$

$$\Rightarrow 1=A(y+1)+B(y-1)\Rightarrow 1=Ay+A+By-B\Rightarrow$$

$$\Rightarrow 1=(A+B)y+A-B\Rightarrow\begin{cases}A+B=0\\ A-B=1\end{cases}\overset{\text{λύνουμε το σύστημα}}{\Rightarrow}\begin{cases}A=\frac{1}{2}\\ B=-\frac{1}{2}\end{cases}$$

Οπότε $\frac{1}{(x-1)(x+1)}=\frac{\frac{1}{2}}{x-1}-\frac{\frac{1}{2}}{x+1}$

Άρα το ζητούμενο ολοκλήρωμα είναι

$$=\int_{\frac{1}{2}}^{0}\frac{1}{y^2-1}dy=\int_{\frac{1}{2}}^{0}\frac{\frac{1}{2}}{x-1}-\frac{\frac{1}{2}}{x+1}dx=\frac{1}{2}\int_{\frac{1}{2}}^{0}\frac{1}{x-1}dx-\frac{1}{2}\int_{\frac{1}{2}}^{0}\frac{1}{x+1}dx=$$

$$=\frac{1}{2}\left[\ln|x-1|\right]_{\frac{1}{2}}^{0}-\frac{1}{2}\left[\ln|x+1|\right]_{\frac{1}{2}}^{0}=$$

$$=\frac{1}{2}\cancel{\ln|0-1|}-\frac{1}{2}\ln\left|\frac{1}{2}-1\right|-\frac{1}{2}\cancel{\ln|0+1|}+\frac{1}{2}\ln\left|\frac{1}{2}+1\right|=$$

$$= -\frac{1}{2}\ln\frac{1}{2} + \frac{1}{2}\ln\frac{3}{2} = \frac{1}{2}\ln\frac{\frac{3}{\cancel{2}}}{\frac{1}{\cancel{2}}} = \frac{\ln 3}{2}$$