Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Β

31.22 1)

α) $\int_0^2 \frac{x-2}{x+1}dx = \int_0^2 \frac{x+1-3}{x+1}dx = \int_0^2 \frac{\cancel{x+1}}{\cancel{x+1}}dx - \int_0^2 \frac{3}{x+1}dx =$

$= 1\cdot(2-0) - 3\left[\ln|x+1|\right]_0^2 = 2 - 3\left[\ln 3 - \ln 1\right]_0^2 = 2 - 3\ln 3$

β) $\int_0^4 \frac{3x-5}{x+2}dx = \int_0^4 \frac{3x+6-11}{x+2}dx = \int_0^4 \frac{3x+6}{x+2}dx - \int_0^4 \frac{11}{x+2}dx =$

$= \int_0^4 \frac{3\cancel{(x+2)}}{\cancel{x+2}}dx - 11\int_0^4 \frac{1}{x+2}dx = 3(4-0) - 11\left[\ln|x+2|\right]_0^4 =$

$= 12 - 11(\ln 6 - \ln 2) = 12 - 11\ln\frac{6}{2} = 12 - 11\ln 3$

31.22 2)

$\int_{-1}^0 \frac{x-1}{x+2}dx = \int_{-1}^0 \frac{x+2-3}{x+2}dx = \int_{-1}^0 \frac{\cancel{x+2}}{\cancel{x+2}}dx - \int_{-1}^0 \frac{3}{x+2}dx =$

$= 1(0-(-1)) - 3\left[\ln|x+2|\right]_{-1}^0 = 1 - 3\left(\ln|0+2| - \cancel{\ln|-1+2|}\right) = 1 - 3\ln 2$

31.22 3)

$\int_0^1 \frac{x+2}{x+1}dx = \int_0^1 \frac{x+1+1}{x+1}dx = \int_0^1 \frac{\cancel{x+1}}{\cancel{x+1}}dx + \int_0^1 \frac{1}{x+1}dx = 1(1-0) + \left[\ln|x+1|\right]_0^1 =$

$= 1 + \ln 2$

31.22 4)

$\int_0^1 \frac{2x+3}{x+1}dx = \int_0^1 \frac{2x+2+1}{x+1}dx = \int_0^1 \frac{2x+2}{x+1}dx + \int_0^1 \frac{1}{x+1}dx =$

$= \int_0^1 \frac{2\cancel{(x+1)}}{\cancel{x+1}}dx + \left[\ln|x+1|\right]_0^1 = 2(1-0) + \ln 2 = 2 + \ln 2$

31.22 5)

$\int_3^4 \frac{2x+1}{x-2}dx = \int_3^4 \frac{2x-4+5}{x-2}dx = \int_3^4 \frac{2x-4}{x-2}dx + \int_3^4 \frac{5}{x-2}dx =$

$= \int_3^4 \frac{2\cancel{(x-2)}}{\cancel{x-2}}dx + 5\left[\ln|x-2|\right]_3^4 = 2(4-3) + 5\cdot\ln 2 = 2 + 5\ln 2$

31.22 6)

$\int_0^1 \frac{x^2}{x+1}dx = \int_0^1 \frac{x^2-1+1}{x+1}dx = \int_0^1 \frac{x^2-1}{x+1}dx + \int_0^1 \frac{1}{x+1}dx =$

$= \int_0^1 \frac{(x-1)\cancel{(x+1)}}{\cancel{x+1}}dx + \left[\ln|x+1|\right]_0^1 = \left[\frac{x^2}{2} - x\right]_0^1 + \ln 2 = -\frac{1}{2} + \ln 2$

31.22 7)

$\int_0^1 \frac{x^3}{x+1}dx = \int_0^1 \frac{x^3+1-1}{x+1}dx = \int_0^1 \frac{x^3+1}{x+1}dx - \int_0^1 \frac{1}{x+1}dx =$

$$=\int_0^1 \frac{\cancel{(x+1)}\left(x^2-x+1\right)}{\cancel{x+1}}dx-\left[\ln|x+1|\right]_0^1=\left[\frac{x^3}{3}-\frac{x^2}{2}+x\right]_0^1-\ln 2=\frac{1}{3}-\frac{1}{2}+1-\ln 2=$$

$$=\frac{5}{6}-\ln 2$$

31.22 8)

$$\int_{-1}^0 \frac{x^2-3x+4}{x^2-3x+2}dx=\int_{-1}^0 \frac{x^2-3x+2+2}{x^2-3x+2}dx=\int_{-1}^0 \frac{\cancel{x^2-3x+2}}{\cancel{x^2-3x+2}}dx+\int_{-1}^0 \frac{2}{x^2-3x+2}dx=$$

$$=\int_{-1}^0 1dx+\int_{-1}^0 \frac{2}{x^2-3x+2}dx=1\left(0-(-1)\right)+\int_{-1}^0 \frac{2}{x^2-3x+2}dx=1+\int_{-1}^0 \frac{2}{x^2-3x+2}dx$$

Οπότε αν I είναι το ζητούμενο ολοκλήρωμα είναι $I=1+\int_{-1}^0 \frac{2}{x^2-3x+2}dx$ (1)

Θα υπολογίσουμε τώρα το ολοκλήρωμα $I_1=\int_{-1}^0 \frac{2}{x^2-3x+2}dx$

Είναι $\frac{2}{x^2-3x+2}\overset{\Delta=1,\ x_{1,2}=\frac{3\pm1}{2}=\begin{cases}\nearrow x_1=\frac{3-1}{2}\Rightarrow x_1=1\\ \searrow x_2=\frac{3+1}{2}\Rightarrow x_2=2\end{cases}}{=}\frac{2}{(x-1)(x-2)}$

Οπότε θα πρέπει να βρούμε τα A και B ώστε $\frac{2}{x^2-3x+2}=\frac{A}{x-1}+\frac{B}{x-2}$

Έχουμε

$$\frac{x+1}{x^2-3x+2}=\frac{A}{x-1}+\frac{B}{x-2}\Rightarrow$$

$$\Rightarrow \cancel{(x-1)(x-2)}\frac{2}{\cancel{(x-1)(x-2)}}=\cancel{(x-1)}(x-2)\frac{A}{\cancel{x-1}}+(x-1)\cancel{(x-2)}\frac{B}{\cancel{x-2}}$$

$$\Rightarrow 2=A(x-2)+B(x-1)\Rightarrow x+1=Ax-2A+Bx-B\Rightarrow$$

$$\Rightarrow 2=(A+B)x+(-2A-B)\Rightarrow \begin{cases}A+B=0\\-2A-B=2\end{cases}\overset{\text{λύνουμε το σύστημα}}{\Rightarrow}\begin{cases}A=-2\\B=2\end{cases}$$

Οπότε $\frac{2}{x^2-3x+2}=\frac{-2}{x-1}+\frac{2}{x-2}$

Άρα

$$I_1=\int_{-1}^0 \frac{x+1}{x^2-3x+2}dx=\int_{-1}^0 \frac{-2}{x-1}+\frac{2}{x-2}dx=-2\int_{-1}^0 \frac{1}{x-1}dx+2\int_{-1}^0 \frac{1}{x-2}dx=$$

$$=-2\left[\ln|x-1|\right]_{-1}^0+2\left[\ln|x-2|\right]_{-1}^0=$$

$$=-2\left(\cancel{\ln|0-1|}-\ln|-1-1|\right)+2\left(\ln|0-2|-\ln|-1-2|\right)=$$

$$= 2\ln 2 + 2(\ln 2 - \ln 3) = 2\ln 2 + 2\ln 2 - 2\ln 3 =$$

$$= 2(2\ln 2 - \ln 3) = 2\ln\frac{4}{3}$$

$$I_1 = \int_{-1}^{0} \frac{2}{x^2 - 3x + 2}dx = 2\ln\frac{4}{3} \quad (2)$$

$$(1),(2) \Rightarrow I = 1 + 2\ln\frac{4}{3}$$