

Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Β

31.19 1)

$$\begin{aligned}\int_0^{\frac{\pi}{4}} \frac{x+2}{\sin^2 x} dx &= \int_0^{\frac{\pi}{4}} (x+2)(\varepsilon\phi x)' dx = \left[(x+2)\varepsilon\phi x \right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} (x+2)' \varepsilon\phi x dx = \\ &= \frac{\pi}{4} + 2 + \int_0^{\frac{\pi}{4}} \frac{-\eta\mu x}{\sin x} dx = \frac{\pi}{4} + 2 + \left[\ln(\sin x) \right]_0^{\frac{\pi}{4}} = \frac{\pi}{4} + 2 + \ln \frac{\sqrt{2}}{2}\end{aligned}$$

31.19 2)

$$\begin{aligned}\int_0^{\frac{\pi}{4}} \frac{x}{\sin^2 x} dx &= \int_0^{\frac{\pi}{4}} x(\varepsilon\phi x)' dx = \left[x\varepsilon\phi x \right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} (x)' \varepsilon\phi x dx = \frac{\pi}{4} - \int_0^{\frac{\pi}{4}} \frac{\eta\mu x}{\sin x} dx = \\ &= \frac{\pi}{4} + \int_0^{\frac{\pi}{4}} \frac{-\eta\mu x}{\sin x} dx = \frac{\pi}{4} + \left[\ln(\sin x) \right]_0^{\frac{\pi}{4}} = \frac{\pi}{4} + \ln \frac{\sqrt{2}}{2} = \frac{\pi + 4 \ln \frac{\sqrt{2}}{2}}{4} = \\ &= \frac{\pi + 4 \ln \frac{4}{16}}{4} = \frac{\pi - 4 \ln 4}{4}\end{aligned}$$

31.19 3)

$$\begin{aligned}\int_0^{\frac{\pi}{4}} \frac{x+3}{\sin^2 x} dx &= \int_0^{\frac{\pi}{4}} (x+3)(\varepsilon\phi x)' dx = \left[(x+3)\varepsilon\phi x \right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} (x+3)' \varepsilon\phi x dx = \\ &= \frac{\pi}{4} + 3 - \int_0^{\frac{\pi}{4}} \frac{\eta\mu x}{\sin x} dx = \frac{\pi}{4} + 3 + \int_0^{\frac{\pi}{4}} \frac{-\eta\mu x}{\sin x} dx = \frac{\pi}{4} + 3 + \left[\ln(\sin x) \right]_0^{\frac{\pi}{4}} = \frac{\pi}{4} + 3 + \ln \frac{\sqrt{2}}{2} = \\ &= \frac{\pi + 12 + 4 \ln \frac{\sqrt{2}}{2}}{4} = \frac{\pi + 12 + 4 \ln \frac{4}{16}}{4} = \frac{\pi + 12 - 4 \ln 4}{4}\end{aligned}$$

31.19 4)

$$\begin{aligned}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{x}{\eta\mu^2 x} dx &= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} x(-\sigma\phi x)' dx = \left[-x\sigma\phi x \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (x)' (-\sigma\phi x) dx = \\ &= \frac{\pi}{4} + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\sigma\upsilon\nu x}{\eta\mu x} dx = \frac{\pi}{4} + \left[\ln(\eta\mu x) \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \frac{\pi}{4} - \ln \frac{\sqrt{2}}{2} = \frac{\pi - 4 \ln \frac{\sqrt{2}}{2}}{4} = \frac{\pi - \ln \frac{4}{14}}{4} \\ &= \frac{\pi + \ln 4}{4}\end{aligned}$$

31.19 5)

$$\begin{aligned}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{x-1}{\eta\mu^2 x} dx &= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (x-1)(-\sigma\phi x)' dx = \left[-(x-1)\sigma\phi x \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (x-1)' (-\sigma\phi x) dx = \\ &= \frac{\pi}{4} - 1 + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\sigma\upsilon\nu x}{\eta\mu x} dx = \frac{\pi}{4} - 1 + \left[\ln(\eta\mu x) \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \frac{\pi}{4} - 1 - \ln \frac{\sqrt{2}}{2} = \frac{\pi - 4 - 4 \ln \frac{\sqrt{2}}{2}}{4} =\end{aligned}$$

$$= \frac{\pi - 4 - \ln \frac{4}{14}}{4} = \frac{\pi - 4 + \ln 4}{4}$$