Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Β

31.18 1)

$$\int_0^1 x\cdot 2^x dx=\int_0^1 x\cdot\left(\frac{2^x}{\ln 2}\right)' dx=\left[\frac{x\cdot 2^x}{\ln 2}\right]_0^1-\int_0^1 (x)'\cdot\frac{2^x}{\ln 2}dx=\frac{2}{\ln 2}-\frac{1}{\ln 2}\int_0^1 2^x dx=$$

$$=\frac{2}{\ln 2}-\frac{1}{\ln 2}\left[\frac{2^x}{\ln 2}\right]_0^1=\frac{2}{\ln 2}-\frac{1}{\ln 2}\left[\frac{2^1}{\ln 2}-\frac{1}{\ln 2}\right]=\frac{2}{\ln 2}-\frac{1}{\ln^2 2}$$

31.18 2)

$$\int_0^{\frac{1}{\ln 3}} x\cdot 3^x dx=\int_0^{\frac{1}{\ln 3}} x\cdot\left(\frac{3^x}{\ln 3}\right)' dx=\left[\frac{x\cdot 3^x}{\ln 3}\right]_0^{\frac{1}{\ln 3}}-\int_0^{\frac{1}{\ln 3}} (x)'\cdot\frac{3^x}{\ln 3}dx=$$

$$=\frac{\frac{1}{\ln 3}\cdot 3^{\frac{1}{\ln 3}}}{\ln 3}-\frac{1}{\ln 3}\int_0^{\frac{1}{\ln 3}} 3^x dx\overset{3^{\frac{1}{\ln 3}}=e^{\ln 3^{\frac{1}{\ln 3}}}=e^{\frac{1}{\ln 3}\ln 3}=e}{=}\frac{e}{\ln^2 3}-\frac{1}{\ln 3}\left[\frac{3^x}{\ln 3}\right]_0^{\frac{1}{\ln 3}}=$$

$$=\frac{e}{\ln^2 3}-\frac{1}{\ln 3}\left(\frac{3^{\frac{1}{\ln 3}}}{\ln 3}-\frac{3^0}{\ln 3}\right)\overset{3^{\frac{1}{\ln 3}}=e^{\ln 3^{\frac{1}{\ln 3}}}=e^{\frac{1}{\ln 3}\ln 3}=e}{=}\frac{e}{\ln^2 3}-\frac{e}{\ln^2 3}+\frac{1}{\ln^2 3}=\frac{1}{\ln^2 3}$$

31.18 3)

$$\int_0^{\frac{1}{\ln 7}} x\cdot 7^x dx=\int_0^{\frac{1}{\ln 7}} x\cdot\left(\frac{7^x}{\ln 7}\right)' dx=\left[\frac{x\cdot 7^x}{\ln 7}\right]_0^{\frac{1}{\ln 7}}-\int_0^{\frac{1}{\ln 7}} (x)'\cdot\frac{7^x}{\ln 7}dx=$$

$$=\frac{\frac{1}{\ln 7}\cdot 7^{\frac{1}{\ln 7}}}{\ln 7}-\frac{1}{\ln 7}\int_0^{\frac{1}{\ln 7}} 7^x dx\overset{7^{\frac{1}{\ln 7}}=e^{\ln 7^{\frac{1}{\ln 7}}}=e^{\frac{1}{\ln 7}\ln 7}=e}{=}\frac{e}{\ln^2 7}-\frac{1}{\ln 7}\left[\frac{7^x}{\ln 7}\right]_0^{\frac{1}{\ln 7}}=$$

$$=\frac{e}{\ln^2 7}-\frac{1}{\ln 7}\left(\frac{3^{\frac{1}{\ln 7}}}{\ln 7}-\frac{3^0}{\ln 7}\right)\overset{7^{\frac{1}{\ln 7}}=e^{\ln 7^{\frac{1}{\ln 7}}}=e^{\frac{1}{\ln 7}\ln 7}=e}{=}\frac{e}{\ln^2 7}-\frac{e}{\ln^2 7}+\frac{1}{\ln^2 7}=\frac{1}{\ln^2 7}$$

31.18 4)

$$\int_{\frac{1}{2}}^{\frac{1}{\ln 5}+\frac{1}{2}}(2x-1)5^x dx=\int_{\frac{1}{2}}^{\frac{1}{\ln 5}+\frac{1}{2}}(2x-1)\cdot\left(\frac{5^x}{\ln 5}\right)' dx=$$

$$=\left[\frac{(2x-1)\cdot 5^x}{\ln 5}\right]_{\frac{1}{2}}^{\frac{1}{\ln 5}+\frac{1}{2}}-\int_{\frac{1}{2}}^{\frac{1}{\ln 5}+\frac{1}{2}}(2x-1)'\cdot\frac{5^x}{\ln 5}dx=$$

$$=\frac{\left(2\frac{1}{\ln 5}+\cancel{2}\frac{1}{\cancel{2}}-1\right)\cdot 5^{\frac{1}{\ln 5}+\frac{1}{2}}}{\ln 5}-0-\frac{2}{\ln 5}\int_{\frac{1}{2}}^{\frac{1}{\ln 5}+\frac{1}{2}}5^x dx$$

$$=\frac{\frac{2\cdot 5^{\frac{1}{\ln 5}+\frac{1}{2}}}{\ln 5}}{\ln 5}-\frac{2}{\ln 5}\left[\frac{5^x}{\ln 5}\right]_{\frac{1}{2}}^{\frac{1}{\ln 5}+\frac{1}{2}}=\frac{2\cdot 5^{\frac{1}{\ln 5}+\frac{1}{2}}}{\ln^2 5}-\frac{2}{\ln 5}\left(\frac{5^{\frac{1}{\ln 5}+\frac{1}{2}}}{\ln 5}-\frac{5^{\frac{1}{2}}}{\ln 5}\right)=$$

$$=\cancel{\frac{2\cdot 5^{\frac{1}{\ln 5}+\frac{1}{2}}}{\ln^2 5}}-\cancel{\frac{2\cdot 5^{\frac{1}{\ln 5}+\frac{1}{2}}}{\ln^2 5}}+\frac{2\cdot 5^{\frac{1}{2}}}{\ln^2 5}=\frac{2\cdot\sqrt{5}}{\ln^2 5}$$

31.18 5)

$$\int_{-\frac{3}{4}}^{\frac{1}{\ln 4}-\frac{3}{4}}(4x+3)4^x dx=\int_{-\frac{3}{4}}^{\frac{1}{\ln 4}-\frac{3}{4}}(4x+3)\cdot\left(\frac{4^x}{\ln 4}\right)' dx=$$

$$=\left[\frac{(4x+3)\cdot 4^x}{\ln 4}\right]_{-\frac{3}{4}}^{\frac{1}{\ln 4}-\frac{3}{4}}-\int_{-\frac{3}{4}}^{\frac{1}{\ln 4}-\frac{3}{4}}(4x+3)'\cdot\frac{4^x}{\ln 4}dx=$$

$$=\frac{\left(4\frac{1}{\ln 4}-\cancel{4}\frac{3}{\cancel{4}}+3\right)\cdot 4^{\frac{1}{\ln 4}-\frac{3}{4}}}{\ln 4}-0-\frac{4}{\ln 4}\int_{-\frac{3}{4}}^{\frac{1}{\ln 4}-\frac{3}{4}}4^x dx$$

$$=\frac{\frac{4\cdot 4^{\frac{1}{\ln 4}-\frac{3}{4}}}{\ln 4}}{\ln 4}-\frac{4}{\ln 4}\left[\frac{4^x}{\ln 4}\right]_{-\frac{3}{4}}^{\frac{1}{\ln 4}-\frac{3}{4}}=\frac{4\cdot 4^{\frac{1}{\ln 4}-\frac{3}{4}}}{\ln^2 4}-\frac{4}{\ln 4}\left(\frac{4^{\frac{1}{\ln 4}-\frac{3}{4}}}{\ln 4}-\frac{4^{-\frac{3}{4}}}{\ln 4}\right)=$$

$$=\cancel{\frac{4\cdot 4^{\frac{1}{\ln 4}-\frac{3}{4}}}{\ln^2 4}}-\cancel{\frac{4\cdot 4^{\frac{1}{\ln 4}-\frac{3}{4}}}{\ln^2 4}}+\frac{4\cdot 4^{-\frac{3}{4}}}{\ln^2 4}\overset{\substack{4^{-\frac{3}{4}}=\left(2^2\right)^{-\frac{3}{4}}=2^{-\frac{3}{2}}=\frac{1}{\sqrt{8}}=\frac{1}{2\sqrt{2}}=\frac{\sqrt{2}}{4}\\ \ln^2 4=(2\ln 2)^2=4\ln^2 4}}{=}\frac{4\sqrt{2}}{4\cdot 4\ln^2 2}=\frac{\sqrt{2}}{4\ln^2 2}$$

31.18 6)

$$\int_0^1 x^2\cdot 5^x dx=\int_0^1 x^2\cdot\left(\frac{5^x}{\ln 5}\right)' dx=\left[\frac{x^2\cdot 5^x}{\ln 5}\right]_0^1-\int_0^1\left(x^2\right)'\frac{5^x}{\ln 5}dx=$$

$$=\frac{5}{\ln 5}-\int_0^1 2x\frac{5^x}{\ln 5}dx=\frac{5}{\ln 5}-\frac{2}{\ln 5}\int_0^1 x\cdot 5^x dx=\frac{5}{\ln 5}-\frac{2}{\ln 5}\int_0^1 x\cdot\left(\frac{5^x}{\ln 5}\right)' dx=$$

$$=\frac{5}{\ln 5}-\frac{2}{\ln 5}\left(\left[\frac{x\cdot 5^x}{\ln 5}\right]_0^1-\int_0^1(x)'\frac{5^x}{\ln 5}dx\right)=$$

$$=\frac{5}{\ln 5}-\frac{2}{\ln 5}\left(\frac{5}{\ln 5}-\frac{1}{\ln 5}\int_0^1 5^x dx\right)=\frac{5}{\ln 5}-\frac{10}{\ln^2 5}+\frac{2}{\ln^2 5}\left[\frac{5^x}{\ln 5}\right]_0^1=$$

$$=\frac{5}{\ln 5}-\frac{10}{\ln^2 5}+\frac{2}{\ln^2 5}\left(\frac{5}{\ln 5}-\frac{1}{\ln 5}\right)=\frac{5}{\ln 5}-\frac{10}{\ln^2 5}+\frac{8}{\ln^3 5}=\frac{8-10\ln 5+5\ln^2 5}{\ln^3 5}$$

31.18 7)

$$\int_0^1\left(x^2-x\right)2^x dx=\int_0^1\left(x^2-x\right)\cdot\left(\frac{2^x}{\ln 2}\right)' dx=\left[\frac{\left(x^2-x\right)\cdot 2^x}{\ln 2}\right]_0^1-\int_0^1\left(x^2-x\right)'\frac{2^x}{\ln 2}dx=$$

$$=0-\int_0^1(2x-1)\frac{2^x}{\ln 2}dx=-\frac{1}{\ln 2}\int_0^1(2x-1)\cdot 2^x dx=-\frac{1}{\ln 2}\int_0^1(2x-1)\cdot\left(\frac{2^x}{\ln 2}\right)' dx=$$

$$=-\frac{1}{\ln 2}\int_0^1 (2x-1)\cdot\left(\frac{2^x}{\ln 2}\right)' dx=-\frac{1}{\ln 2}\left(\left[\frac{(2x-1)2^x}{\ln 2}\right]_0^1-\int_0^1 (2x-1)'\frac{2^x}{\ln 2}dx\right)=$$

$$=-\frac{1}{\ln 2}\left(\left(\frac{2}{\ln 2}-\frac{(-1)2^0}{\ln 2}\right)-\int_0^1 2\cdot\frac{2^x}{\ln 2}dx\right)=$$

$$=-\frac{1}{\ln 2}\left(\frac{3}{\ln 2}-\frac{2}{\ln 2}\int_0^1 2^x dx\right)=-\frac{3}{\ln^2 2}+\frac{2}{\ln^2 2}\left[\frac{2^x}{\ln 2}\right]_0^1=-\frac{3}{\ln^2 2}+\frac{2}{\ln^2 2}\left(\frac{2^1}{\ln 2}-\frac{2^0}{\ln 2}\right)=$$

$$=-\frac{3}{\ln^2 2}+\frac{2}{\ln^3 2}=\frac{2-3\ln 2}{\ln^3 2}$$