Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Β

31.16 1)

α) $\int_0^{\pi} x\eta\mu x dx = \int_0^{\pi} x(-\sigma\upsilon\nu x)' dx = [-x\sigma\upsilon\nu x]_0^{\pi} - \int_0^{\pi} (x)'(-\sigma\upsilon\nu x)dx =$

$= -\pi\sigma\upsilon\nu\pi + \int_0^{\pi} \sigma\upsilon\nu x dx = \pi + [\eta\mu x]_0^{\pi} = \pi + 0 - 0 = \pi$

β) $\int_0^{\frac{\pi}{2}} x^2\sigma\upsilon\nu x dx = \int_0^{\frac{\pi}{2}} x^2(\eta\mu x)' dx = \left[x^2\eta\mu x\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} (x^2)' \eta\mu x dx =$

$= \frac{\pi^2}{4} - \int_0^{\frac{\pi}{2}} 2x\eta\mu x dx = \frac{\pi^2}{4} - 2\int_0^{\frac{\pi}{2}} x(-\sigma\upsilon\nu x)' dx =$

$= \frac{\pi^2}{4} - 2\left([-x\sigma\upsilon\nu x]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} (x)'(-\sigma\upsilon\nu x)dx\right) = \frac{\pi^2}{4} - 2\left(0 + \int_0^{\frac{\pi}{2}} \sigma\upsilon\nu x dx\right) =$

$= \frac{\pi^2}{4} - 2\int_0^{\frac{\pi}{2}} \sigma\upsilon\nu x dx = \frac{\pi^2}{4} - 2[\eta\mu x]_0^{\frac{\pi}{2}} = \frac{\pi^2}{4} - 2\cdot 1 = \frac{\pi^2}{4} - 2$

31.16 2)

$\int_0^{\frac{\pi}{2}} x\eta\mu x dx = \int_0^{\frac{\pi}{2}} x(-\sigma\upsilon\nu x)' dx = [-x\sigma\upsilon\nu x]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} (x)'(-\sigma\upsilon\nu x)dx =$

$= 0 + \int_0^{\frac{\pi}{2}} \sigma\upsilon\nu x dx = [\eta\mu x]_0^{\frac{\pi}{2}} = 1 - 0 = 1$

31.16 3)

$\int_0^{\pi} x\sigma\upsilon\nu x dx = \int_0^{\pi} x(\eta\mu x)' dx = [x\eta\mu x]_0^{\pi} - \int_0^{\pi} (x)' \eta\mu x dx =$

$= 0 - \int_0^{\pi} \eta\mu x dx = -[-\sigma\upsilon\nu x]_0^{\pi} = [\sigma\upsilon\nu x]_0^{\pi} = -1 - 1 = -2$

31.16 4)

$\int_0^{\frac{\pi}{2}} x\sigma\upsilon\nu x\, dx = \int_0^{\frac{\pi}{2}} x(\eta\mu x)' dx = [x\eta\mu x]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} (x)' \eta\mu x dx =$

$= \frac{\pi}{2} - \int_0^{\frac{\pi}{2}} \eta\mu x dx = \frac{\pi}{2} - [-\sigma\upsilon\nu x]_0^{\frac{\pi}{2}} = \frac{\pi}{2} + [\sigma\upsilon\nu x]_0^{\frac{\pi}{2}} = \frac{\pi}{2} - 1$

31.16 5)

$\int_0^{2\pi} x\eta\mu x\, dx = \int_0^{2\pi} x(-\sigma\upsilon\nu x)' dx = [-x\sigma\upsilon\nu x]_0^{2\pi} - \int_0^{2\pi} (x)'(-\sigma\upsilon\nu x)dx =$

$= -2\pi + \int_0^{2\pi} \sigma\upsilon\nu x dx = -2\pi + [\eta\mu x]_0^{2\pi} = -2\pi$

31.16 6)

$\int_{\frac{\pi}{2}}^{0} (3x+5)\eta\mu x\, dx = \int_{\frac{\pi}{2}}^{0} (3x+5)(-\sigma\upsilon\nu x)' dx$

$= \left[-(3x+5)\sigma\upsilon\nu x\right]_{\frac{\pi}{2}}^{0} - \int_{\frac{\pi}{2}}^{0} (3x+5)'(-\sigma\upsilon\nu x)dx = -5 + \int_{\frac{\pi}{2}}^{0} 3\sigma\upsilon\nu x dx =$

$$=-5+\left[3ημx\right]_{\frac{\pi}{2}}^{0}=-5-3=-8$$

31.16 7)

$$\int_{\frac{\pi}{2}}^{0} xσυν(2x)\,dx=\int_{\frac{\pi}{2}}^{0} x\left(\frac{ημ2x}{2}\right)' dx=\left[\frac{xημ2x}{2}\right]_{\frac{\pi}{2}}^{0}-\int_{\frac{\pi}{2}}^{0}(x)'\,\frac{ημ2x}{2}dx=$$

$$=0-\frac{1}{2}\int_{\frac{\pi}{2}}^{0} ημ2xdx=-\frac{1}{2}\left[\frac{-συν2x}{2}\right]_{\frac{\pi}{2}}^{0}=\frac{1}{4}\left[συν2x\right]_{\frac{\pi}{2}}^{0}=\frac{1}{4}(1+1)=\frac{1}{2}$$

31.16 8)

$$\int_{0}^{\frac{\pi}{6}} xημ(3x)\,dx=\int_{0}^{\frac{\pi}{6}} x\left(-\frac{συν3x}{3}\right)' dx=\left[\frac{-xσυν3x}{3}\right]_{0}^{\frac{\pi}{6}}-\int_{0}^{\frac{\pi}{6}}(x)'\left(-\frac{συν3x}{3}\right)dx=$$

$$=0+\frac{1}{3}\int_{0}^{\frac{\pi}{6}} συν3xdx=\frac{1}{3}\left[\frac{ημ3x}{3}\right]_{0}^{\frac{\pi}{6}}=\frac{1}{3}\left(\frac{1}{3}-0\right)=\frac{1}{9}$$

31.16 9)

$$\int_{\pi}^{2\pi} xσυν3x\,dx=\int_{\pi}^{2\pi} x\left(\frac{ημ3x}{3}\right)' dx=\left[\frac{xημ3x}{3}\right]_{\pi}^{2\pi}-\int_{\pi}^{2\pi}(x)'\,\frac{ημ3x}{3}dx=$$

$$=0-\frac{1}{3}\int_{\pi}^{2\pi} ημ3xdx=-\frac{1}{3}\left[\frac{-συν3x}{3}\right]_{\pi}^{2\pi}=\frac{1}{9}\left[συν3x\right]_{\pi}^{2\pi}=\frac{1}{9}(1+1)=\frac{2}{9}$$

31.16 10)

$$\int_{\pi}^{2\pi} x\,ημ3x\,dx=\int_{\pi}^{2\pi} x\left(-\frac{συν3x}{3}\right)' dx=\left[\frac{-xσυν3x}{3}\right]_{\pi}^{2\pi}-\int_{\pi}^{2\pi}(x)'\left(-\frac{συν3x}{3}\right)dx=$$

$$=\frac{-2\pi-\pi}{3}+\frac{1}{3}\int_{\pi}^{2\pi} συν3xdx=-\pi+\frac{1}{3}\left[\frac{ημ3x}{3}\right]_{\pi}^{2\pi}=-\pi+0=-\pi$$

31.16 11)

$$\int_{0}^{\pi} xημ\frac{x}{2}dx=\int_{0}^{\pi} x\left(-\frac{συν\frac{x}{2}}{\frac{1}{2}}\right)' dx=\left[-2xσυν\frac{x}{2}\right]_{0}^{\pi}-\int_{0}^{\pi}(x)'\left(-2συν\frac{x}{2}\right)dx=$$

$$=0+2\int_{0}^{\pi} συν\frac{x}{2}dx=2\left[\frac{ημ\frac{x}{2}}{\frac{1}{2}}\right]_{0}^{\pi}=2\left[2ημ\frac{x}{2}\right]_{0}^{\pi}=2(2-0)=4$$

31.16 12)

$$\int_{0}^{\pi}(2x-1)συν(3x)\,dx=\int_{0}^{\pi}(2x-1)\left(\frac{ημ3x}{3}\right)' dx=$$

$$=\left[\frac{(2x-1)ημ3x}{3}\right]_{0}^{\pi}-\int_{0}^{\pi}(2x-1)'\,\frac{ημ3x}{3}dx=0-\frac{2}{3}\int_{0}^{\pi} ημ3xdx=-\frac{2}{3}\left[\frac{-συν3x}{3}\right]_{0}^{\pi}=$$

$$=\frac{2}{9}\left[συν3x\right]_{0}^{\pi}=\frac{2}{9}(-1-1)=-\frac{4}{9}$$

31.16 13)

$$\int_{-\frac{1}{3}}^{\frac{\pi-2}{6}}(x-2)\eta\mu(3x+1)dx=\int_{-\frac{1}{3}}^{\frac{\pi-2}{6}}(x-2)\left(-\frac{\sigma\upsilon\nu(3x+1)}{3}\right)'dx=$$

$$=\left[\frac{-(x-2)\sigma\upsilon\nu(3x+1)}{3}\right]_{-\frac{1}{3}}^{\frac{\pi-2}{6}}-\int_{-\frac{1}{3}}^{\frac{\pi-2}{6}}(x)'\left(-\frac{\sigma\upsilon\nu(3x+1)}{3}\right)dx=$$

$$=\left(\frac{-\left(\frac{\pi-2}{6}-2\right)\sigma\upsilon\nu\left(\frac{\pi-2}{2}+1\right)}{3}-\frac{-\left(-\frac{1}{3}-2\right)\sigma\upsilon\nu 0}{3}\right)+\frac{1}{3}\int_{-\frac{1}{3}}^{\frac{\pi-2}{6}}\sigma\upsilon\nu(3x+1)dx=$$

$$=\left(0-\frac{\frac{7}{3}}{3}\right)+\frac{1}{3}\int_{-\frac{1}{3}}^{\frac{\pi-2}{6}}\sigma\upsilon\nu(3x+1)dx=-\frac{7}{9}+\frac{1}{3}\left[\frac{\eta\mu(3x+1)}{3}\right]_{-\frac{1}{3}}^{\frac{\pi-2}{6}}=$$

$$=-\frac{7}{9}+\frac{1}{3}\left(\frac{\eta\mu\left(\frac{\pi-2}{2}+1\right)}{3}-\frac{\eta\mu\left(-3\frac{1}{3}+1\right)}{3}\right)=-\frac{7}{9}+\frac{1}{3}\left(\frac{1}{3}-0\right)=-\frac{6}{9}=-\frac{2}{3}$$

31.16 14)

$$\int_{3\pi}^{0}(x-1)\sigma\upsilon\nu\frac{x}{3}dx=\int_{3\pi}^{0}(x-1)\left(\frac{\eta\mu\frac{x}{3}}{\frac{1}{3}}\right)'dx=$$

$$=\left[\frac{(x-1)\eta\mu\frac{x}{3}}{\frac{1}{3}}\right]_{3\pi}^{0}-\int_{3\pi}^{0}(x-1)'\frac{\eta\mu\frac{x}{3}}{\frac{1}{3}}dx=0-3\int_{3\pi}^{0}\eta\mu\frac{x}{3}dx=-3\left[-\frac{\sigma\upsilon\nu\frac{x}{3}}{\frac{1}{3}}\right]_{3\pi}^{0}=$$

$$=9\left[\sigma\upsilon\nu\frac{x}{3}\right]_{3\pi}^{0}=9(1+1)=18$$

31.16 15)

$$\int_{0}^{\pi}x^2\sigma\upsilon\nu x dx=\int_{0}^{\pi}x^2(\eta\mu x)'dx=\left[x^2\eta\mu x\right]_0^{\pi}-\int_0^{\pi}(x^2)'\eta\mu x dx=$$

$$=0-\int_0^{\pi}2x\eta\mu x dx=-2\int_0^{\pi}x(-\sigma\upsilon\nu x)'dx=-2\left(\left[-x\sigma\upsilon\nu x\right]_0^{\pi}-\int_0^{\pi}(x)'(-\sigma\upsilon\nu x)dx\right)=$$

$$=-2\pi-2\int_0^{\pi}\sigma\upsilon\nu x dx=-2\pi-2\left[\eta\mu x\right]_0^{\pi}=-2\pi-2\cdot 0=-2\pi$$

31.16 16)

$$\int_0^{\pi}x^2\eta\mu x\,dx=\int_0^{\pi}x^2(-\sigma\upsilon\nu x)'dx=\left[-x^2\sigma\upsilon\nu x\right]_0^{\pi}-\int_0^{\pi}(x^2)'(-\sigma\upsilon\nu x)dx=$$

$$=\pi^2+\int_0^{\pi}2x\sigma\upsilon\nu x dx=\pi^2+\int_0^{\pi}2x(\eta\mu x)'dx=\pi^2+\left[2x\eta\mu x\right]_0^{\pi}-\int_0^{\pi}(2x)'\eta\mu x dx=$$

$$=\pi^2+0-2\int_0^{\pi}\eta\mu x dx=\pi^2-2[-\sigma\upsilon\nu x]_0^{\pi}=\pi^2-2(1+1)=\pi^2-4$$

31.16 17)

$$\int_0^{\frac{\pi}{2}}x^2\eta\mu x\,dx=\int_0^{\frac{\pi}{2}}x^2(-\sigma\upsilon\nu x)'dx=\left[-x^2\sigma\upsilon\nu x\right]_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}(x^2)'(-\sigma\upsilon\nu x)dx=$$

$$=0+\int_0^{\frac{\pi}{2}}2x\sigma\upsilon\nu xdx=\int_0^{\frac{\pi}{2}}2x(\eta\mu x)'dx=[2x\eta\mu x]_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}(2x)'\eta\mu xdx=$$

$$=\pi-2\int_0^{\frac{\pi}{2}}\eta\mu xdx=\pi-2[-\sigma\upsilon\nu x]_0^{\frac{\pi}{2}}=\pi-2(0+1)=\pi-2$$

31.16 18)

$$\int_0^{\pi}x^2\sigma\upsilon\nu(2x)dx=\int_0^{\pi}x^2\left(\frac{\eta\mu 2x}{2}\right)'dx=\left[\frac{x^2\eta\mu 2x}{2}\right]_0^{\pi}-\int_0^{\pi}(x^2)'\frac{\eta\mu 2x}{2}dx=$$

$$=0-\int_0^{\pi}\cancel{2}x\frac{\eta\mu 2x}{\cancel{2}}dx=-\int_0^{\pi}x\left(-\frac{\sigma\upsilon\nu 2x}{2}\right)'dx=$$

$$=-\left(\left[-\frac{x\sigma\upsilon\nu 2x}{2}\right]_0^{\pi}-\int_0^{\pi}(x)'\left(-\frac{\sigma\upsilon\nu 2x}{2}\right)dx\right)=-\left(-\frac{\pi}{2}+\frac{1}{2}\int_0^{\pi}\sigma\upsilon\nu 2xdx\right)=$$

$$=\frac{\pi}{2}-\frac{1}{2}\int_0^{\pi}\sigma\upsilon\nu xdx=\frac{\pi}{2}-\frac{1}{2}[\eta\mu x]_0^{\pi}=\frac{\pi}{2}-\frac{1}{2}0=\frac{\pi}{2}$$

31.16 19)

$$\int_0^{\frac{\pi}{2}}(x^2-3x+1)\eta\mu x\,dx=\int_0^{\frac{\pi}{2}}(x^2-3x+1)(-\sigma\upsilon\nu x)'dx=$$

$$=\left[-(x^2-3x+1)\sigma\upsilon\nu x\right]_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}(x^2-3x+1)'(-\sigma\upsilon\nu x)dx=1+\int_0^{\frac{\pi}{2}}(2x-3)\sigma\upsilon\nu xdx=$$

$$=1+\int_0^{\frac{\pi}{2}}(2x-3)(\eta\mu x)'dx=1+\left[(2x-3)\eta\mu x\right]_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}(2x-3)'\eta\mu xdx=$$

$$=1+\pi-3-2\int_0^{\frac{\pi}{2}}\eta\mu xdx=\pi-2-2[-\sigma\upsilon\nu x]_0^{\frac{\pi}{2}}=\pi-2-2(0+1)=\pi-4$$

31.16 20)

$$\int_{\frac{\pi}{2}}^{2\pi}(x^2+1)\eta\mu 3x\,dx=\int_{\frac{\pi}{2}}^{2\pi}(x^2+1)\left(-\frac{\sigma\upsilon\nu 3x}{3}\right)'dx=$$

$$=\left[-\frac{(x^2+1)\sigma\upsilon\nu 3x}{3}\right]_{\frac{\pi}{2}}^{2\pi}-\int_{\frac{\pi}{2}}^{2\pi}(x^2+1)'\left(-\frac{\sigma\upsilon\nu 3x}{3}\right)dx=-\frac{4\pi^2+1}{3}+\frac{1}{3}\int_{\frac{\pi}{2}}^{2\pi}2x\sigma\upsilon\nu 3xdx=$$

$$=-\frac{4\pi^2+1}{3}+\frac{1}{3}\int_{\frac{\pi}{2}}^{2\pi}2x\left(\frac{\eta\mu 3x}{3}\right)'dx=-\frac{4\pi^2+1}{3}+\frac{1}{3}\left(\left[\frac{2x\eta\mu 3x}{3}\right]_{\frac{\pi}{2}}^{2\pi}-\int_{\frac{\pi}{2}}^{2\pi}(2x)'\frac{\eta\mu 3x}{3}dx\right)=$$

$$=-\frac{4\pi^2+1}{3}+\frac{1}{3}\left(\frac{\pi}{3}-\frac{2}{3}\int_{\frac{\pi}{2}}^{2\pi}\eta\mu 3x dx\right)=-\frac{4\pi^2+1}{3}+\frac{\pi}{9}-\frac{2}{9}\int_{\frac{\pi}{2}}^{2\pi}\eta\mu 3x dx=$$

$$=-\frac{4\pi^2+1}{3}+\frac{\pi}{9}-\frac{2}{9}\left[-\frac{\sigma\upsilon\nu 3x}{3}\right]_{\frac{\pi}{2}}^{2\pi}=-\frac{4\pi^2+1}{3}+\frac{\pi}{9}-\frac{2}{9}\left(-\frac{1}{3}\right)=-\frac{4\pi^2+1}{3}+\frac{\pi}{9}+\frac{2}{27}=$$

$$=\frac{-36\pi^2+3\pi-7}{27}$$

31.16 21)

$$\int_0^{2\pi}x^2\sigma\upsilon\nu\frac{x}{2}dx=\int_0^{2\pi}x^2\left(\frac{\eta\mu\frac{x}{2}}{\frac{1}{2}}\right)'dx=\left[\frac{x^2\eta\mu\frac{x}{2}}{\frac{1}{2}}\right]_0^{2\pi}-\int_0^{2\pi}\left(x^2\right)'\frac{\eta\mu\frac{x}{2}}{\frac{1}{2}}dx=$$

$$=0-\int_0^{2\pi}4x\eta\mu\frac{x}{2}dx=-4\int_0^{2\pi}x\left(-\frac{\sigma\upsilon\nu\frac{x}{2}}{\frac{1}{2}}\right)'dx=$$

$$=-4\left(\left[-\frac{x\sigma\upsilon\nu\frac{x}{2}}{\frac{1}{2}}\right]_0^{2\pi}-\int_0^{2\pi}(x)'\left(-\frac{\sigma\upsilon\nu\frac{x}{2}}{\frac{1}{2}}\right)dx\right)=$$

$$=-4\left(4\pi+2\int_0^{2\pi}\sigma\upsilon\nu\frac{x}{2}dx\right)=-16\pi-8\left[\frac{\eta\mu\frac{x}{2}}{\frac{1}{2}}\right]_0^{2\pi}=-16\pi$$

31.16 22)

$$\int_0^{\frac{\pi}{2}}x^3\eta\mu 2x\,dx=\int_0^{\frac{\pi}{2}}x^3\left(-\frac{\sigma\upsilon\nu 2x}{2}\right)'dx=\left[-\frac{x^3\sigma\upsilon\nu 2x}{2}\right]_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}\left(x^3\right)'\left(-\frac{\sigma\upsilon\nu 2x}{2}\right)dx=$$

$$=-\frac{\left(\frac{\pi}{2}\right)^3\sigma\upsilon\nu\cancel{2}\frac{\pi}{\cancel{2}}}{2}+\frac{3}{2}\int_0^{\frac{\pi}{2}}x^2\sigma\upsilon\nu 2xdx=\frac{\pi^3}{16}+\frac{3}{2}\int_0^{\frac{\pi}{2}}x^2\left(\frac{\eta\mu 2x}{2}\right)'dx=$$

$$=\frac{\pi^3}{16}+\frac{3}{2}\left(\left[x^2\left(\frac{\eta\mu 2x}{2}\right)\right]_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}\left(x^2\right)'\frac{\eta\mu 2x}{2}dx\right)=\frac{\pi^3}{16}+\frac{3}{2}\left(0-\int_0^{\frac{\pi}{2}}\cancel{2}x\frac{\eta\mu 2x}{\cancel{2}}dx\right)=$$

$$=\frac{\pi^3}{16}-\frac{3}{2}\int_0^{\frac{\pi}{2}}x\eta\mu 2xdx=\frac{\pi^3}{16}-\frac{3}{2}\int_0^{\frac{\pi}{2}}x\left(-\frac{\sigma\upsilon\nu 2x}{2}\right)dx=$$

$$=\frac{\pi^3}{16}-\frac{3}{2}\left(\left[-\frac{x\sigma\upsilon\nu 2x}{2}\right]_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}(x)'\left(-\frac{\sigma\upsilon\nu 2x}{2}\right)dx\right)=$$

$$= \frac{\pi^3}{16} - \frac{3}{2}\left(-\frac{\frac{\pi}{2}\sigma\upsilon\nu\cancel{2}\frac{\pi}{\cancel{2}}}{2} + \frac{1}{2}\int_0^{\frac{\pi}{2}} \sigma\upsilon\nu 2x dx \right) = \frac{\pi^3}{16} - \frac{3}{2}\left(\frac{\pi}{4} + \frac{1}{2}\int_0^{\frac{\pi}{2}} \sigma\upsilon\nu 2x dx \right) =$$

$$= \frac{\pi^3}{16} - \frac{3\pi}{8} - \frac{3}{4}\left[\frac{\eta\mu 2x}{2} \right]_0^{\frac{\pi}{2}} = \frac{\pi^3}{16} - \frac{3\pi}{8} - \frac{3}{4}\cdot 0 = \frac{\pi^3}{16} - \frac{3\pi}{8}$$