31.15 1)

α) $$\int_0^2 x^2e^x dx=\int_0^2 x^2\left(e^x\right)'dx=\left[x^2e^x\right]_0^2-\int_0^2\left(x^2\right)'\left(e^x\right)dx=4e^2-\int_0^2 2xe^x dx=$$

$$=4e^2-2\int_0^2 x\left(e^x\right)'dx=4e^2-2\left(\left[xe^x\right]_0^2-\int_0^2(x)'\left(e^x\right)dx\right)=$$

$$=4e^2-2\left(2e^2-\int_0^2 e^x dx\right)=\cancel{4e^2}-\cancel{4e^2}+2\left[e^x\right]_0^2=2\left(e^2-1\right)=2e^2-2$$

β) $$\int_0^1 x^2e^{-2x}dx=\int_0^1 x^2\left(\frac{e^{-2x}}{-2}\right)'dx=\left[\frac{x^2e^{-2x}}{-2}\right]_0^1-\int_0^1\left(x^2\right)'\left(\frac{e^{-2x}}{-2}\right)dx=$$

$$=\frac{e^{-2}}{-2}-\int_0^1\cancel{2}x\frac{e^{-2x}}{-\cancel{2}}dx=\frac{e^{-2}}{-2}+\int_0^1 xe^{-2x}dx=\frac{e^{-2}}{-2}+\int_0^1 x\left(\frac{e^{-2x}}{-2}\right)'dx=$$

$$=\frac{e^{-2}}{-2}+\left[\frac{xe^{-2x}}{-2}\right]_0^1-\int_0^1(x)'\frac{e^{-2x}}{-2}dx=\frac{e^{-2}}{-2}+\frac{e^{-2}}{-2}+\frac{1}{2}\int_0^1 e^{-2x}dx=$$

$$=-e^{-2}+\frac{1}{2}\int_0^1 e^{-2x}dx=-e^{-2}+\frac{1}{2}\left[\frac{e^{-2x}}{-2}\right]_0^1=-e^{-2}+\frac{1}{2}\left(\frac{e^{-2}}{-2}-\frac{1}{-2}\right)=\frac{1}{4}-\frac{5e^{-2}}{4}$$

γ) $$\int_0^1\left(x^2-x\right)e^x dx=\int_0^1\left(x^2-x\right)\left(e^x\right)'dx=\left[\left(x^2-x\right)e^x\right]_0^1-\int_0^1\left(x^2-x\right)'\left(e^x\right)dx=$$

$$=0-\int_0^1(2x-1)e^x dx=-\int_0^1(2x-1)\left(e^x\right)'dx=$$

$$=-\left[(2x-1)e^x\right]_0^1+\int_0^1(2x-1)'e^x dx=-\left[(2x-1)e^x\right]_0^1+\int_0^1(2x-1)'e^x dx=$$

$$=-(e+1)+2\int_0^1 e^x dx=-e-1+2\left[e^x\right]_0^1=-e-1+2(e-1)=e-3$$

31.15 2)

$$\int_0^1 x^2e^x dx=\int_0^1 x^2\left(e^x\right)'dx=\left[x^2e^x\right]_0^1-\int_0^1\left(x^2\right)'\left(e^x\right)dx=e-\int_0^1 2xe^x dx=$$

$$=e-2\int_0^1 x\left(e^x\right)'dx=e-2\left(\left[xe^x\right]_0^1-\int_0^1(x)'\left(e^x\right)dx\right)=e-2\left(e-\int_0^1 e^x dx\right)=$$

$$=e-2e+2\left[e^x\right]_0^1=-e+2(e-1)=-e+2e-2=e-2$$

31.15 3)

$$\int_1^2 x^2e^{2x}dx=\int_1^2 x^2\left(\frac{e^{2x}}{2}\right)'dx=\left[\frac{x^2e^{2x}}{2}\right]_1^2-\int_1^2\left(x^2\right)'\left(\frac{e^{2x}}{2}\right)dx=$$

$$=\frac{4e^4-e^2}{2}-\int_1^2\cancel{2}x\frac{e^{2x}}{\cancel{2}}dx=\frac{4e^4-e^2}{2}-\int_1^2 xe^{2x}dx=\frac{4e^4-e^2}{2}-\int_1^2 x\left(\frac{e^{2x}}{2}\right)'dx=$$

$$=\frac{4e^4-e^2}{2}-\left(\left[\frac{xe^{2x}}{2}\right]_1^2-\int_1^2(x)'\frac{e^{2x}}{2}dx\right)=$$

$$=\frac{4e^4-e^2}{2}-\left(\frac{2e^4-e^2}{2}-\frac{1}{2}\int_1^2 e^{2x}dx\right)=\frac{4e^4-e^2}{2}-\frac{2e^4-e^2}{2}+\frac{1}{2}\int_1^2 e^{2x}dx=$$

$$=\frac{4e^4-\cancel{e^2}-2e^4+\cancel{e^2}}{2}+\frac{1}{2}\left[\frac{e^{2x}}{2}\right]_1^2=e^4+\frac{1}{2}\left(\frac{e^4}{2}-\frac{e^2}{2}\right)=e^4+\frac{e^4-e^2}{4}=\frac{5e^4-e^2}{4}$$

31.15 4)

$$\int_0^1 x^2e^{2x}dx=\int_0^1 x^2\left(\frac{e^{2x}}{2}\right)'dx=\left[\frac{x^2e^{2x}}{2}\right]_0^1-\int_0^1\left(x^2\right)'\left(\frac{e^{2x}}{2}\right)dx=\frac{e^2}{2}-\int_0^1\cancel{2}x\frac{e^{2x}}{\cancel{2}}dx=$$

$$=\frac{e^2}{2}-\int_0^1 xe^{2x}dx=\frac{e^2}{2}-\int_0^1 x\left(\frac{e^{2x}}{2}\right)'dx=\frac{e^2}{2}-\left(\left[\frac{xe^{2x}}{2}\right]_0^1-\int_0^1(x)'\frac{e^{2x}}{2}dx\right)=$$

$$=\frac{e^2}{2}-\left(\frac{e^2}{2}-\frac{1}{2}\int_0^1 e^{2x}dx\right)=\frac{\cancel{e^2}}{\cancel{2}}-\frac{\cancel{e^2}}{\cancel{2}}+\frac{1}{2}\int_0^1 e^{2x}dx=\frac{1}{2}\left[\frac{e^{2x}}{2}\right]_0^1=\frac{1}{2}\left(\frac{e^2}{2}-\frac{e^0}{2}\right)=$$

$$=\frac{e^2-1}{4}$$

31.15 5)

$$\int_{-1}^1 x^2e^{-x}dx=\int_{-1}^1 x^2\left(-e^{-x}\right)'dx=\left[-x^2e^{-x}\right]_{-1}^1-\int_{-1}^1\left(x^2\right)'\left(-e^{-x}\right)dx=$$

$$=-e^{-1}+e+\int_{-1}^1 2xe^{-x}dx=-\frac{1}{e}+e+\int_{-1}^1 2x\left(-e^{-x}\right)'dx=$$

$$=-\frac{1}{e}+e+\left[-2xe^{-x}\right]_{-1}^1-\int_{-1}^1(2x)'\left(-e^{-x}\right)dx=-\frac{1}{e}+e-2e^{-1}-2e+2\int_{-1}^1 e^{-x}dx=$$

$$=-\frac{3}{e}-e+2\left[-e^{-x}\right]_{-1}^1=-\frac{3}{e}-e+2\left(-e^{-1}+e^1\right)=-\frac{3}{e}-e-\frac{2}{e}+2e=e-\frac{5}{e}$$

31.15 6)

$$\int_0^1\left(x^2+1\right)e^xdx=\int_0^1\left(x^2+1\right)\left(e^x\right)'dx=\left[\left(x^2+1\right)e^x\right]_0^1-\int_0^1\left(x^2+1\right)'\left(e^x\right)dx=$$

$$=2e-e^0-\int_0^1 2xe^xdx=2e-1-2\int_0^1 x\left(e^x\right)'dx=2e-1-2\left(\left[xe^x\right]_0^1-\int_0^1(x)'\left(e^x\right)dx\right)=$$

$$=2e-1-2\left(e-\int_0^1 e^xdx\right)=\cancel{2e}-1-\cancel{2e}+2\left[e^x\right]_0^1=-1+2(e-1)=$$

$$=-1+2e-2=2e-3$$

31.15 7)

$$\int_1^2\left(3x^2-2x+1\right)e^xdx=\int_1^2\left(3x^2-2x+1\right)\left(e^x\right)'dx=$$

$$=\left[\left(3x^2-2x+1\right)e^x\right]_1^2-\int_1^2\left(3x^2-2x+1\right)'e^xdx=$$

$$=9e^2-2e-\int_1^2(6x-2)e^xdx=9e^2-2e-\int_1^2(6x-2)\left(e^x\right)'dx=$$

$$=9e^2-2e-\left(\left[(6x-2)e^x\right]_1^2-\int_1^2(6x-2)'e^xdx\right)=$$

$$=9e^2-2e-\left(10e^2-4e-\int_1^2 6e^xdx\right)=9e^2-2e-10e^2+4e+6\int_1^2 e^xdx=$$

$$=-e^2+2e+6\left[e^x\right]_1^2=-e^2+2e+6\left(e^2-e\right)=-e^2+2e+6e^2-6e=5e^2-4e$$

31.15 8)

$$\int_1^{-1}\left(x^2+2x\right)e^x dx=\int_1^{-1}\left(x^2+2x\right)\left(e^x\right)' dx=$$

$$=\left[\left(x^2+2x\right)e^x\right]_1^{-1}-\int_1^{-1}\left(x^2+2x\right)' e^x dx=-e^{-1}-3e-\int_1^{-1}(2x+2)e^x dx=$$

$$=-\frac{1}{e}-3e-\int_1^{-1}(2x+2)\left(e^x\right)' dx=-\frac{1}{e}-3e-\left(\left[(2x+2)e^x\right]_1^{-1}-\int_1^{-1}(2x+2)' e^x dx\right)=$$

$$=-\frac{1}{e}-3e-\left(-4e-\int_1^{-1}2e^x dx\right)=-\frac{1}{e}-3e+4e+2\int_1^{-1}e^x dx=$$

$$=-\frac{1}{e}+e+2\left[e^x\right]_1^{-1}=-\frac{1}{e}+e+2\left(e^{-1}-e\right)=-\frac{1}{e}+e+\frac{2}{e}-2e=\frac{1}{e}-e$$

31.15 9)

$$\int_0^{-1}\frac{x^2}{e^{3x}}dx=\int_0^{-1}x^2e^{-3x}dx=\int_0^{-1}x^2\left(\frac{e^{-3x}}{-3}\right)' dx=\left[\frac{x^2e^{-3x}}{-3}\right]_0^{-1}-\int_0^{-1}\left(x^2\right)'\left(\frac{e^{-3x}}{-3}\right)dx=$$

$$=-\frac{e^3}{3}-\int_0^{-1}2x\left(\frac{e^{-3x}}{-3}\right)dx=-\frac{e^3}{3}+\frac{2}{3}\int_0^{-1}xe^{-3x}dx=-\frac{e^3}{3}+\frac{2}{3}\int_0^{-1}x\left(\frac{e^{-3x}}{-3}\right)' dx=$$

$$=-\frac{e^3}{3}+\frac{2}{3}\left(\left[\frac{xe^{-3x}}{-3}\right]_0^{-1}-\int_0^{-1}(x)'\frac{e^{-3x}}{-3}dx\right)=-\frac{e^3}{3}+\frac{2}{3}\left(\frac{-e^3}{-3}+\frac{1}{3}\int_0^{-1}e^{-3x}dx\right)=$$

$$=-\frac{e^3}{3}+\frac{2e^3}{9}+\frac{2}{9}\int_0^{-1}e^{-3x}dx=-\frac{e^3}{9}+\frac{2}{9}\left[\frac{e^{-3x}}{-3}\right]_0^{-1}=$$

$$=-\frac{e^3}{9}+\frac{2}{9}\left(\frac{e^3}{-3}-\frac{e^0}{-3}\right)=-\frac{e^3}{9}-\frac{2e^3}{27}+\frac{2}{27}=\frac{-5e^3+2}{27}$$

31.15 10)

$$\int_0^{-1}\left(x^2-2x+5\right)e^{-x}dx=\int_0^{-1}\left(x^2-2x+5\right)\left(-e^{-x}\right)' dx=$$

$$=\left[-\left(x^2-2x+5\right)e^{-x}\right]_0^{-1}-\int_0^{-1}\left(x^2-2x+5\right)'\left(-e^{-x}\right)dx=$$

$$=-8e+5+\int_0^{-1}(2x-2)e^{-x}dx=-8e+5+\int_0^{-1}(2x-2)\left(-e^{-x}\right)' dx=$$

$$=-8e+5+\left[(2x-2)\left(-e^{-x}\right)\right]_0^{-1}-\int_0^{-1}(2x-2)'\left(-e^{-x}\right)dx=$$

$$=-8e+5+4e-2+\int_0^{-1}2e^{-x}dx=-4e+3+\left[-2e^{-x}\right]_0^{-1}=$$

$$=-4e+3-2e+2=5-6e$$

31.15 11)

$$\int_0^1 x^3 e^{2x}dx = \int_0^1 x^3\left(\frac{e^{2x}}{2}\right)' dx = \left[\frac{x^3 e^{2x}}{2}\right]_0^1 - \int_0^1 \left(x^3\right)'\left(\frac{e^{2x}}{2}\right)dx =$$

$$= \frac{e^2}{2} - \int_0^1 3x^2\,\frac{e^{2x}}{2}dx = \frac{e^2}{2} - \frac{3}{2}\int_0^1 x^2 e^{2x}dx = \frac{e^2}{2} - \frac{3}{2}\int_0^1 x^2\left(\frac{e^{2x}}{2}\right)' dx =$$

$$= \frac{e^2}{2} - \frac{3}{2}\left(\left[\frac{x^2 e^{2x}}{2}\right]_0^1 - \int_0^1 \left(x^2\right)'\,\frac{e^{2x}}{2}dx\right) = \frac{e^2}{2} - \frac{3}{2}\left(\frac{e^2}{2} - \int_0^1 \cancel{2}x\,\frac{e^{2x}}{\cancel{2}}dx\right) =$$

$$= \frac{e^2}{2} - \frac{3e^2}{4} + \frac{3}{2}\int_0^1 xe^{2x}dx = -\frac{e^2}{4} + \frac{3}{2}\int_0^1 x\left(\frac{e^{2x}}{2}\right)' dx =$$

$$= -\frac{e^2}{4} + \frac{3}{2}\left(\left[\frac{xe^{2x}}{2}\right]_0^1 - \int_0^1 (x)'\,\frac{e^{2x}}{2}dx\right) = -\frac{e^2}{4} + \frac{3}{2}\left(\frac{e^2}{2} - \frac{1}{2}\int_0^1 e^{2x}dx\right) =$$

$$= -\frac{e^2}{4} + \frac{3e^2}{4} - \frac{3}{4}\int_0^1 e^{2x}dx = \frac{\cancel{2}e^2}{\cancel{4}\,2} - \frac{3}{4}\left[\frac{e^{2x}}{2}\right]_0^1 = \frac{e^2}{2} - \frac{3}{4}\left(\frac{e^2}{2} - \frac{1}{2}\right) = \frac{e^2}{2} - \frac{3e^2}{8} + \frac{3}{8}$$

$$= \frac{e^2+3}{8}$$