Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Β

31.14 1)

α) $$\int_{-1}^{0} xe^x dx = \int_{-1}^{0} x\left(e^x\right)' dx = \left[xe^x\right]_{-1}^{0} - \int_{-1}^{0}(x)'\left(e^x\right)dx = e^{-1} - \int_{-1}^{0} e^x dx =$$

$$= e^{-1} - \left[e^x\right]_{-1}^{0} = e^{-1} - \left(1 - e^{-1}\right) = -1 + 2e^{-1}$$

β) $$\int_0^1 (3x+1)e^x dx = \int_0^1 (3x+1)\left(e^x\right)' dx = \left[(3x+1)e^x\right]_0^1 - \int_0^1 (3x+1)'\left(e^x\right)dx =$$

$$= 4e - 1 - \int_0^1 3e^x dx = 4e - 1 - \left[3e^x\right]_0^1 = 4e - 1 - 3e + 3 = e + 2$$

γ) $$\int_0^{\frac{2}{3}} xe^{3x}dx = \int_0^{\frac{2}{3}} x\left(\frac{e^{3x}}{3}\right)' dx = \left[\frac{xe^{3x}}{3}\right]_0^{\frac{2}{3}} - \int_0^{\frac{2}{3}}(x)'\left(\frac{e^{3x}}{3}\right)dx =$$

$$= \frac{2e^2}{9} - \frac{1}{3}\int_0^{\frac{2}{3}} e^{3x}dx = \frac{2e^2}{9} - \frac{1}{3}\left[\frac{e^{3x}}{3}\right]_0^{\frac{2}{3}} = \frac{2e^2}{9} - \frac{1}{3}\left(\frac{e^2}{3} - \frac{1}{3}\right) = \frac{e^2+1}{9}$$

δ) $$\int_0^{\frac{1}{2}} \frac{x}{e^{2x}}dx = \int_0^{\frac{1}{2}} xe^{-2x}dx = \int_0^{\frac{1}{2}} x\left(\frac{e^{-2x}}{-2}\right)' dx = \left[\frac{xe^{-2x}}{-2}\right]_0^{\frac{1}{2}} - \int_0^{\frac{1}{2}}(x)'\left(\frac{e^{-2x}}{-2}\right)dx =$$

$$= \frac{e^{-1}}{-4} + \frac{1}{2}\int_0^{\frac{1}{2}} e^{-2x}dx = -\frac{e^{-1}}{4} + \frac{1}{2}\left[\frac{e^{-2x}}{-2}\right]_0^{\frac{1}{2}} = -\frac{e^{-1}}{4} + \frac{1}{2}\left(\frac{e^{-1}}{-2} - \frac{1}{-2}\right) = \frac{1}{4} - \frac{1}{2e}$$

31.14 2)

$$\int_0^1 xe^x dx = \int_0^1 x\left(e^x\right)' dx = \left[xe^x\right]_0^1 - \int_0^1 (x)'\left(e^x\right)dx = e - \int_0^1 e^x dx =$$

$$= e - \left[e^x\right]_0^1 = e - (e-1) = 1$$

31.14 3)

$$\int_1^2 xe^x dx = \int_1^2 x\left(e^x\right)' dx = \left[xe^x\right]_1^2 - \int_1^2 (x)'\left(e^x\right)dx = \left(2e^2 - e\right) - \int_1^2 e^x dx =$$

$$= \left(2e^2 - e\right) - \left[e^x\right]_1^2 = \left(2e^2 - e\right) - \left(e^2 - e\right) = 2e^2 - \cancel{e} - e^2 + \cancel{e} = \boxed{e^2}$$

31.14 4)

$$\int_0^{-1} (x+1)e^x dx = \int_0^{-1} (x+1)\left(e^x\right)' dx = \left[(x+1)e^x\right]_0^{-1} - \int_0^{-1} (x+1)'\left(e^x\right)dx =$$

$$= -1 - \int_0^{-1} e^x dx = -1 - \left[e^x\right]_0^{-1} = -1 - \left(e^{-1} - e^0\right) = -1 - \frac{1}{e} + 1 = -\frac{1}{e}$$

31.14 5)

$$\int_{\frac{1}{2}}^1 (2x+1)e^x dx = \int_{\frac{1}{2}}^1 (2x+1)\left(e^x\right)' dx = \left[(2x+1)e^x\right]_{\frac{1}{2}}^1 - \int_{\frac{1}{2}}^1 (2x+1)'\left(e^x\right)dx =$$

$$= 3e - 2\sqrt{e} - \int_{\frac{1}{2}}^1 2e^x dx = 3e - 2\sqrt{e} - \left[2e^x\right]_{\frac{1}{2}}^1 = 3e - \cancel{2\sqrt{e}} - 2e + \cancel{2\sqrt{e}} = e$$

31.14 6)

$$\int_{\frac{5}{3}}^2 (3x-2)e^x dx = \int_{\frac{5}{3}}^2 (3x-2)\left(e^x\right)' dx = \left[(3x-2)e^x\right]_{\frac{5}{3}}^2 - \int_{\frac{5}{3}}^2 (3x-2)'\left(e^x\right)dx =$$

$$=4e^2-3e^{\frac{5}{3}}-\int_{\frac{5}{3}}^{2}3e^x dx=4e^2-3e^{\frac{5}{3}}-\left[3e^x\right]_{\frac{5}{3}}^{2}=4e^2-\cancel{3e^{\frac{5}{3}}}-3e^2+\cancel{3e^{\frac{5}{3}}}=e^2$$

31.14 7)

$$\int_0^{\frac{1}{2}}xe^{2x}dx=\int_0^{\frac{1}{2}}x\left(\frac{e^{2x}}{2}\right)'dx=\left[\frac{xe^{2x}}{2}\right]_0^{\frac{1}{2}}-\int_0^{\frac{1}{2}}(x)'\left(\frac{e^{2x}}{2}\right)dx=$$

$$=\frac{e}{4}-\frac{1}{2}\int_0^{\frac{1}{2}}e^{2x}dx=\frac{e}{4}-\frac{1}{2}\left[\frac{e^{2x}}{2}\right]_0^{\frac{1}{2}}=\frac{e}{4}-\frac{1}{2}\left(\frac{e}{2}-\frac{e^0}{2}\right)=\cancel{\frac{e}{4}}-\cancel{\frac{e}{4}}+\frac{1}{4}=\frac{1}{4}$$

31.14 8)

$$\int_0^{\frac{1}{3}}xe^{3x}dx=\int_0^{\frac{1}{3}}x\left(\frac{e^{3x}}{3}\right)'dx=\left[\frac{xe^{3x}}{3}\right]_0^{\frac{1}{3}}-\int_0^{\frac{1}{3}}(x)'\left(\frac{e^{3x}}{3}\right)dx=$$

$$=\frac{e}{9}-\frac{1}{3}\int_0^{\frac{1}{3}}e^{3x}dx=\frac{e}{9}-\frac{1}{3}\left[\frac{e^{3x}}{3}\right]_0^{\frac{1}{3}}=\frac{e}{9}-\frac{1}{3}\left(\frac{e}{3}-\frac{e^0}{3}\right)=\cancel{\frac{e}{9}}-\cancel{\frac{e}{9}}+\frac{1}{9}=\frac{1}{9}$$

31.14 9)

$$\int_{-1}^{1}xe^{-x}dx=\int_{-1}^{1}x\left(-e^{-x}\right)'dx=\left[-xe^{-x}\right]_{-1}^{1}-\int_{-1}^{1}(x)'\left(-e^{-x}\right)dx=$$

$$=-e^{-1}-e+\int_{-1}^{1}e^{-x}dx=-\frac{1}{e}-e+\left[-e^{-x}\right]_{-1}^{1}=-\frac{1}{e}-e+\left(-e^{-1}+e^{1}\right)=$$

$$=-\frac{1}{e}-\cancel{e}-\frac{1}{e}+\cancel{e}=-\frac{2}{e}$$

31.14 10)

$$\int_0^{-\frac{2}{3}}\frac{x}{e^{3x}}dx=\int_0^{-\frac{2}{3}}xe^{-3x}dx=\int_0^{-\frac{2}{3}}x\left(\frac{e^{-3x}}{-3}\right)'dx=\left[\frac{xe^{-3x}}{-3}\right]_0^{-\frac{2}{3}}-\int_0^{-\frac{2}{3}}(x)'\left(\frac{e^{-3x}}{-3}\right)dx=$$

$$=\frac{2e^2}{9}+\frac{1}{3}\int_0^{-\frac{2}{3}}e^{-3x}dx=\frac{2e^2}{9}+\frac{1}{3}\left[\frac{e^{-3x}}{-3}\right]_0^{-\frac{2}{3}}=\frac{2e^2}{9}+\frac{1}{3}\left(\frac{e^2}{-3}-\frac{e^0}{-3}\right)=\frac{2e^2}{9}-\frac{e^2}{9}+\frac{1}{9}=$$

$$=\frac{e^2+1}{9}$$

31.14 11)

$$\int_0^{\frac{1}{2}}(2x-1)e^{-2x}dx=\int_0^{\frac{1}{2}}(2x-1)\left(\frac{e^{-2x}}{-2}\right)'dx=$$

$$=\left[\frac{(2x-1)e^{-2x}}{-2}\right]_0^{\frac{1}{2}}-\int_0^{\frac{1}{2}}(2x-1)'\left(\frac{e^{-2x}}{-2}\right)dx=-\frac{(-1)e^0}{-2}+\int_0^{\frac{1}{2}}e^{-2x}dx=$$

$$=-\frac{1}{2}+\left[\frac{e^{-2x}}{-2}\right]_0^{\frac{1}{2}}=-\frac{1}{2}+\left(\frac{e^{-1}}{-2}-\frac{e^0}{-2}\right)=-\cancel{\frac{1}{2}}-\frac{1}{2e}+\cancel{\frac{1}{2}}=-\frac{1}{2e}$$