

Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Β

31.13 1)

$$\int_0^1 (2x+1)4^{x^2+x} dx = \int_0^2 4^y dy = \left[\frac{4^y}{\ln 4} \right]_0^2 = \frac{4^2}{\ln 4} - \frac{4^0}{\ln 4} = \frac{15}{\ln 4}$$

θέτουμε $y=x^2+x \Rightarrow dy=(2x+1)dx$
 για $x=0 \Rightarrow y=0$
 για $x=1 \Rightarrow y=2$

31.13 2)

$$\int_1^0 x \cdot 3^{x^2} dx = \int_1^0 \frac{1}{2} \cdot 3^y dy = \frac{1}{2} \left[\frac{3^y}{\ln 3} \right]_1^0 = \frac{1}{2} \left(\frac{3^0}{\ln 3} - \frac{3^1}{\ln 3} \right) = \frac{1}{2} \frac{-2}{\ln 3} = -\frac{1}{\ln 3}$$

θέτουμε $y=x^2 \Rightarrow dy=2x dx \Rightarrow x dx = \frac{1}{2} dy$
 για $x=1 \Rightarrow y=1$
 για $x=0 \Rightarrow y=0$

31.13 3)

$$\int_0^{\frac{\pi}{2}} 5^{\eta \mu x} \sigma \upsilon \nu x dx = \int_0^1 5^y dy = \left[\frac{5^y}{\ln 5} \right]_0^1 = \frac{5^1}{\ln 5} - \frac{5^0}{\ln 5} = \frac{4}{\ln 5}$$

θέτουμε $y=\eta \mu x \Rightarrow dy=\sigma \upsilon \nu x dx$
 για $x=\frac{\pi}{2} \Rightarrow y=1$
 για $x=0 \Rightarrow y=0$

31.13 4)

$$\int_1^{\frac{1}{2}} \frac{7^{\frac{1}{x}}}{x^2} dx = -\int_1^2 7^y dy = -\left[\frac{7^y}{\ln 7} \right]_1^2 = -\frac{7^2}{\ln 7} + \frac{7^1}{\ln 7} = \frac{-42}{\ln 7}$$

θέτουμε $y=\frac{1}{x} \Rightarrow dy=-\frac{1}{x^2} dx \Rightarrow \frac{1}{x^2} dx = -dy$
 για $x=1 \Rightarrow y=1$
 για $x=\frac{1}{2} \Rightarrow y=2$

31.13 5)

$$\int_{\frac{1}{2\pi}}^{\frac{2}{\pi}} \frac{e^{\eta \mu \frac{1}{x}} \sigma \upsilon \nu \frac{1}{x}}{x^2} dx = -\int_0^1 e^y dy = -\left[e^y \right]_0^1 = -e^1 + e^0 = 1 - e$$

θέτουμε $y=\eta \mu \frac{1}{x} \Rightarrow dy=-\frac{\sigma \upsilon \nu \frac{1}{x}}{x^2} dx \Rightarrow \frac{\sigma \upsilon \nu \frac{1}{x}}{x^2} dx = -dy$
 για $x=\frac{1}{2\pi} \Rightarrow y=\eta \mu 2\pi=0$
 για $x=\frac{2}{\pi} \Rightarrow y=\eta \mu \frac{\pi}{2}=1$

31.13 6)

$$\int_0^1 (x+1)e^x \sigma \upsilon \nu (x e^x) dx = \int_0^e \sigma \upsilon \nu y dy = [\eta \mu y]_0^e = \eta \mu e$$

θέτουμε $y=x e^x \Rightarrow dy=(x+1)e^x dx$
 για $x=0 \Rightarrow y=0$
 για $x=1 \Rightarrow y=e$

31.13 7)

$$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \sigma \upsilon \nu (x + \sigma \upsilon \nu x) \cdot (1 - \eta \mu x) dx = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \sigma \upsilon \nu y dy = [\eta \mu y]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} = \eta \mu \frac{3\pi}{2} - \eta \mu \frac{\pi}{2} = -1 - 1 = -2$$

θέτουμε $y=x + \sigma \upsilon \nu x \Rightarrow dy=(1 - \eta \mu x) dx$
 για $x=\frac{\pi}{2} \Rightarrow y=\frac{\pi}{2}$
 για $x=\frac{3\pi}{2} \Rightarrow y=\frac{3\pi}{2}$

31.13 8)

$$\int_1^2 e^{x \ln x} (\ln x + 1) dx = \int_0^{\ln 4} e^y dy = \left[e^y \right]_0^{\ln 4} = e^{\ln 4} - e^0 = 4 - 1 = 3$$

θέτουμε $y=x \ln x \Rightarrow dy=(\ln x + 1) dx$
 για $x=1 \Rightarrow y=0$
 για $x=2 \Rightarrow y=2 \ln 2 = \ln 4$

31.13 9)

$$\begin{array}{l} \text{Θέτουμε } y = \frac{x}{e^x} \Rightarrow dy = \frac{1-x}{e^x} dx \\ \text{για } x=0 \Rightarrow y=0 \\ \text{για } x=1 \Rightarrow y=\frac{1}{e} \end{array} \quad \int_0^1 \frac{1-x}{e^x} \sin\left(\frac{x}{e^x}\right) dx = \int_0^{\frac{1}{e}} \sin y dy = [\eta\mu y]_0^{\frac{1}{e}} = \eta\mu \frac{1}{e}$$

31.13 10)

$$\begin{array}{l} \text{Θέτουμε } y = x\eta\mu x \Rightarrow dy = (\eta\mu x + x\sigma\upsilon\nu x) dx \\ \text{για } x = \frac{\pi}{6} \Rightarrow y = \frac{\pi}{12} \\ \text{για } x = \frac{\pi}{2} \Rightarrow y = \frac{\pi}{2} \end{array} \quad \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\eta\mu x + x\sigma\upsilon\nu x}{x\eta\mu x} dx = \int_{\frac{\pi}{12}}^{\frac{\pi}{2}} \frac{1}{y} dy = [\ln y]_{\frac{\pi}{12}}^{\frac{\pi}{2}} =$$

$$= \ln \frac{\pi}{2} - \ln \frac{\pi}{12} = \ln \frac{\cancel{\pi}^{\pi}}{\cancel{\pi}_{12}} = \ln 6$$

31.13 11)

$$\begin{array}{l} \text{Θέτουμε } y = x^x = e^{\ln x^x} = e^{x \ln x} \Rightarrow \\ \Rightarrow dy = e^{x \ln x} (\ln x + 1) dx \\ \Rightarrow dy = x^x (\ln x + 1) dx \\ \text{για } x=1 \Rightarrow y=1 \\ \text{για } x=3 \Rightarrow y=27 \end{array} \quad \int_1^3 x^x (\ln x + 1) dx = \int_1^{27} 1 dy = [y]_1^{27} = 27 - 1 = 26$$