

Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Β

31.12 1)

$$\begin{aligned} & \text{Θέτουμε } y=1+\ln x \Rightarrow dy=\frac{1}{x}dx \\ & \text{για } x=1 \Rightarrow y=1 \\ & \text{για } x=e \Rightarrow y=2 \\ & \int_1^e \frac{dx}{x(1+\ln x)^4} = \int_1^2 \frac{1}{y^4} dy = \left[\frac{y^{-3}}{-3} \right]_1^2 = -\frac{1}{3}(2^{-3}-1^{-3}) = \\ & = -\frac{1}{3}\left(\frac{1}{8}-1\right) = -\frac{1}{3}\frac{-7}{8} = \frac{7}{24} \end{aligned}$$

31.12 2)

$$\begin{aligned} & \text{Θέτουμε } y=\ln x \Rightarrow dy=\frac{1}{x}dx \\ & \text{για } x=e \Rightarrow y=\ln e=1 \\ & \text{για } x=e^3 \Rightarrow y=\ln e^3=3 \\ & \int_e^{e^3} \frac{1}{x \ln x} dx = \int_1^3 \frac{1}{y} dy = [\ln y]_1^3 = \ln 3 - \ln 1 = \ln 3 \end{aligned}$$

31.12 3)

$$\begin{aligned} & \text{Θέτουμε } y=1+3\ln x \Rightarrow dy=3\frac{1}{x}dx \Rightarrow \frac{1}{x}dx=\frac{1}{3}dy \\ & \text{για } x=1 \Rightarrow y=1+3\ln 1=1 \\ & \text{για } x=e \Rightarrow y=1+3\ln e=1+3=4 \\ & \int_1^e \frac{\sqrt{1+3\ln x}}{x} dx = \int_1^4 \frac{1}{3}\sqrt{y} dy = \frac{1}{3} \left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^4 = \\ & = \frac{1}{3} \frac{1}{3} \left(4^{\frac{3}{2}} - 1^{\frac{3}{2}} \right) = \frac{2}{9} (8-1) = \frac{14}{9} \end{aligned}$$

31.12 4)

$$\begin{aligned} & \text{Θέτουμε } y=\ln x \Rightarrow dy=\frac{1}{x}dx \\ & \text{για } x=1 \Rightarrow y=\ln 1=0 \\ & \text{για } x=e^{\frac{\pi}{3}} \Rightarrow y=\ln e^{\frac{\pi}{3}}=\frac{\pi}{3} \\ & \int_1^{e^{\frac{\pi}{3}}} \frac{2\eta\mu(\ln x)}{x} dx = \int_0^{\frac{\pi}{3}} 2\eta\mu y dy = 2[-\sigma\upsilon\nu y]_0^{\frac{\pi}{3}} = \\ & = 2\left(-\sigma\upsilon\nu \frac{\pi}{3} + \sigma\upsilon\nu 0\right) = 2\left(-\frac{1}{2} + 1\right) = 2\frac{1}{2} = 1 \end{aligned}$$

31.12 5)

$$\begin{aligned} & \text{Θέτουμε } y=\ln x+3 \Rightarrow dy=\frac{1}{x}dx \\ & \text{για } x=e \Rightarrow y=\ln e+3=1+3=4 \\ & \text{για } x=e^5 \Rightarrow y=\ln e^5+3=5+3=8 \\ & \int_e^{e^5} \frac{1}{x(\ln x+3)} dx = \int_4^8 \frac{1}{y} dy = [\ln y]_4^8 = \ln 8 - \ln 4 = \ln \frac{8}{4} = \ln 2 \end{aligned}$$

31.12 6)

$$\begin{aligned} & \text{Θέτουμε } y=\ln x \Rightarrow dy=\frac{1}{x}dx \\ & \text{για } x=e \Rightarrow y=1 \\ & \text{για } x=e^2 \Rightarrow y=2 \\ & \int_e^{e^2} \frac{1}{x \ln^2 x} dx = \int_1^2 \frac{1}{y^2} dy = \left[-\frac{1}{y} \right]_1^2 = -\frac{1}{2} + 1 = \frac{1}{2} \end{aligned}$$

31.12 7)

$$\begin{aligned} & \text{Θέτουμε } y=\ln x \Rightarrow dy=\frac{1}{x}dx \\ & \text{για } x=e \Rightarrow y=1 \\ & \text{για } x=e^3 \Rightarrow y=3 \\ & \int_e^{e^3} \frac{\ln x + 1}{x \ln x} dx = \int_1^3 \frac{y+1}{y} dy = \int_1^3 \left(1 + \frac{1}{y} \right) dy = [y + \ln y]_1^3 = \\ & = 3 + \ln 3 - 1 - \ln 1 = 2 + \ln 3 \end{aligned}$$

31.12 8)

$$\int_{e^3}^1 \frac{\ln^2 x}{x} dx \stackrel{\substack{\text{θετούμε } y=\ln x \Rightarrow dy=\frac{1}{x}dx \\ \text{για } x=e^3 \Rightarrow y=3 \\ \text{για } x=1 \Rightarrow y=0}}{=} \int_3^0 y^2 dy = \left[\frac{y^3}{3} \right]_3^0 = \frac{0^3}{3} - \frac{3^3}{3} = -9$$

31.12 9)

$$\int_1^e \frac{\sqrt{1+\ln x}}{2x} dx \stackrel{\substack{\text{θετούμε } y=1+\ln x \Rightarrow dy=\frac{1}{x}dx \\ \text{για } x=1 \Rightarrow y=1+\ln 1=1 \\ \text{για } x=e \Rightarrow y=1+\ln e=2}}{=} \int_1^2 \frac{\sqrt{y}}{2} dy = \frac{1}{2} \left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^2 = \frac{1}{\cancel{2}} \frac{1}{\cancel{3}} \left(2^{\frac{3}{2}} - 1^{\frac{3}{2}} \right) =$$

$$= \frac{1}{3} (\sqrt{8} - 1) = \frac{1}{3} (2\sqrt{2} - 1)$$