

Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Β

31.11 1)

$$\text{1) α) } \int_{\ln \frac{3}{4}}^0 e^x \sqrt{1-e^x} dx = - \int_{\frac{1}{4}}^0 \sqrt{y} dy = \int_0^{\frac{1}{4}} y^{\frac{1}{2}} dy = \left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^{\frac{1}{4}} = \frac{1}{3} \left(\left(\frac{1}{4} \right)^{\frac{3}{2}} - 0^{\frac{3}{2}} \right) = \frac{1}{3} \frac{1}{8} = \frac{1}{24}$$

$$\text{β) } \int_0^{\ln^2 3} \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = \int_0^{\ln^2 3} 2e^y dy = 2 \left[e^y \right]_0^{\ln^2 3} = 2(e^{\ln^2 3} - e^0) = 2(3-1) = 4$$

$$\text{γ) } \int_{\ln 2}^{\ln 7} \frac{2}{1+e^{-2x}} dx = \int_{\ln 2}^{\ln 7} \frac{2}{1+\frac{1}{e^{2x}}} dx = \int_{\ln 2}^{\ln 7} \frac{2e^{2x}}{e^{2x}+1} dx = \int_5^{50} \frac{1}{y} dy = [\ln y]_5^{50} = \ln 50 - \ln 5 = \ln \frac{50}{5} = \ln 10$$

31.11 2)

$$\int_0^1 x e^{x^2+1} dx = \int_1^2 \frac{1}{2} e^y dy = \frac{1}{2} [e^y]_1^2 = \frac{1}{2} (e^2 - e)$$

31.11 3)

$$\int_0^1 x^2 e^{x^3+1} dx = \int_1^2 \frac{1}{3} e^y dy = \frac{1}{3} [e^y]_1^2 = \frac{1}{3} (e^2 - e)$$

31.11 4)

$$\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = \int_1^2 2e^y dy = 2 [e^y]_1^2 = 2(e^2 - e)$$

31.11 5)

$$\int_1^2 e^{x \ln 2} dx = \frac{1}{\ln 2} \int_{\ln 2}^{\ln 4} e^y dy = \frac{1}{\ln 2} [e^y]_{\ln 2}^{\ln 4} = \frac{1}{\ln 2} (e^{\ln 4} - e^{\ln 2}) = \frac{1}{\ln 2} (4 - 2) = \frac{2}{\ln 2}$$

31.11 6)

$$\int_1^0 2x e^{x^2} dx = \int_1^0 e^y dy = [e^y]_1^0 = e^0 - e = 1 - e$$

31.11 7)

$$\begin{array}{l} \text{θέτουμε } y=x^3 \Rightarrow dy=3x^2 dx \Rightarrow x^2 dx = \frac{1}{3} dy \\ \text{για } x=-1 \Rightarrow y=-1 \\ \text{για } x=1 \Rightarrow y=1 \end{array} \quad \int_{-1}^1 6x^2 e^{x^3} dx = \int_{-1}^1 6 \frac{1}{3} e^y dy = 2 \left[e^y \right]_{-1}^1 = 2(e^1 - e^{-1}) = 2e - \frac{2}{e}$$

31.11 8)

$$\begin{array}{l} \text{θέτουμε } y=x^2+x \Rightarrow dy=(2x+1)dx \\ \text{για } x=0 \Rightarrow y=0 \\ \text{για } x=-1 \Rightarrow y=0 \end{array} \quad \int_0^{-1} (2x+1) e^{x^2+x} dx = \int_0^0 e^y dy = 0$$

31.11 9)

$$\begin{array}{l} \text{θέτουμε } y=\frac{1}{x} \Rightarrow dy=-\frac{1}{x^2} dx \Rightarrow \frac{1}{x^2} dx = -dy \\ \text{για } x=2 \Rightarrow y=\frac{1}{2} \\ \text{για } x=1 \Rightarrow y=1 \end{array} \quad \int_2^1 \frac{1}{x^2} e^x dx = -\int_{\frac{1}{2}}^1 e^y dy = \int_1^{\frac{1}{2}} e^y dy = \left[e^y \right]_1^{\frac{1}{2}} = e^{\frac{1}{2}} - e = \sqrt{e} - e$$

31.11 10)

$$\begin{array}{l} \text{θέτουμε } y=2x^2+1 \Rightarrow dy=4x dx \Rightarrow x dx = \frac{1}{4} dy \\ \text{για } x=-1 \Rightarrow y=3 \\ \text{για } x=1 \Rightarrow y=3 \end{array} \quad \int_{-1}^1 x e^{2x^2+1} dx = \int_3^3 \frac{1}{4} e^y dy = 0$$

31.11 11)

$$\begin{array}{l} \text{θέτουμε } y=e^x \Rightarrow dy=e^x dx \\ \text{για } x=\ln \frac{\pi}{3} \Rightarrow y=\frac{\pi}{3} \\ \text{για } x=\ln 2\pi \Rightarrow y=2\pi \end{array} \quad \int_{\ln \frac{\pi}{3}}^{\ln 2\pi} e^x \eta \mu (e^x) dx = \int_{\frac{\pi}{3}}^{2\pi} \eta \mu y dy = \left[-\sigma \upsilon \nu x \right]_{\frac{\pi}{3}}^{2\pi} = -\sigma \upsilon \nu 2\pi + \sigma \upsilon \nu \frac{\pi}{3} =$$

$$= -1 + \frac{1}{2} = -\frac{1}{2}$$

31.11 12)

$$\int_0^{\ln 3} \frac{1}{1+e^{-x}} dx = \int_0^{\ln 3} \frac{1}{1+\frac{1}{e^x}} dx = \int_0^{\ln 3} \frac{1}{\frac{e^x+1}{e^x}} dx = \int_0^{\ln 3} \frac{e^x}{e^x+1} dx =$$

$$\begin{array}{l} \text{θέτουμε } y=e^x+1 \Rightarrow dy=e^x dx \\ \text{για } x=0 \Rightarrow y=e^0+1=2 \\ \text{για } x=\ln 3 \Rightarrow y=e^{\ln 3}+1 \end{array} \quad = \int_2^{e^{\ln 3}+1} \frac{1}{y} dy = \left[\ln y \right]_2^{e^{\ln 3}+1} = \ln 4 - \ln 2 = \ln \frac{4}{2} = \ln 2$$

31.11 13)

$$\begin{array}{l} \text{θέτουμε } y=\eta \mu x \Rightarrow dy=\sigma \upsilon \nu x dx \\ \text{για } x=\frac{\pi}{2} \Rightarrow y=1 \\ \text{για } x=2\pi \Rightarrow y=0 \end{array} \quad \int_{\frac{\pi}{2}}^{2\pi} \sigma \upsilon \nu x e^{\eta \mu x} dx = \int_1^0 e^y dy = \left[e^y \right]_1^0 = e^0 - e^1 = 1 - e$$

31.11 14)

$$\begin{array}{l} \text{θέτουμε } y=\sigma \upsilon \nu x \Rightarrow dy=-\eta \mu x dx \\ \text{για } x=\frac{\pi}{2} \Rightarrow y=0 \\ \text{για } x=0 \Rightarrow y=1 \end{array} \quad \int_{\frac{\pi}{2}}^0 e^{\sigma \upsilon \nu x} \eta \mu x dx = -\int_0^1 e^y dy = \int_1^0 e^y dy = \left[e^y \right]_1^0 = e^0 - e^1 = 1 - e$$

31.11 15)

$$\begin{array}{l} \text{θέτουμε } y=\eta \mu x \Rightarrow dy=\sigma \upsilon \nu x dx \\ \text{για } x=0 \Rightarrow y=0 \\ \text{για } x=\frac{\pi}{2} \Rightarrow y=1 \end{array} \quad \int_0^{\pi/2} e^{\eta \mu x} \sigma \upsilon \nu x dx = \int_0^1 e^y dy = \left[e^y \right]_0^1 = e^1 - e^0 = e - 1$$

