Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Β

31.10 1)

α) $\int_0^{\frac{\pi}{2}} \eta\mu^5 x \cdot \sigma\upsilon\nu x dx$ $\begin{matrix}\text{θέτουμε } y=\eta\mu x \Rightarrow dy=\sigma\upsilon\nu x dx\\ \text{για } x=0 \Rightarrow y=0\\ \text{για } x=\frac{\pi}{2} \Rightarrow y=1\end{matrix}$ $= \int_0^1 y^5 dy = \left[\frac{y^6}{6}\right]_0^1 = \frac{1}{6}$

β) $\int_{\frac{\pi}{6}}^0 \frac{\eta\mu x}{\sigma\upsilon\nu^3 x} dx$ $\begin{matrix}\text{θέτουμε } y=\sigma\upsilon\nu x \Rightarrow dy=-\eta\mu x dx\\ \text{για } x=\frac{\pi}{6} \Rightarrow y=\frac{\sqrt{3}}{2}\\ \text{για } x=0 \Rightarrow y=1\end{matrix}$ $= -\int_{\frac{\sqrt{3}}{2}}^1 \frac{1}{y^3} dy = \int_1^{\frac{\sqrt{3}}{2}} y^{-3} dy = \left[\frac{y^{-2}}{-2}\right]_1^{\frac{\sqrt{3}}{2}} =$

$$= -\frac{1}{2}\left[\left(\frac{\sqrt{3}}{2}\right)^{-2} - 1^{-2}\right] = -\frac{1}{2}\left(\frac{4}{3}-1\right) = -\frac{1}{6}$$

γ) $\int_0^{\frac{\pi}{2}} \frac{\eta\mu 2x}{1+\eta\mu^2 x} dx$ $\begin{matrix}\text{θέτουμε } y=1+\eta\mu^2 x \Rightarrow dy=2\eta\mu x\sigma\upsilon\nu x dx\\ \eta\mu 2x=2\eta\mu x\sigma\upsilon\nu x\\ \Rightarrow \quad dy=\eta\mu 2x dx\\ \text{για } x=0 \Rightarrow y=1\\ \text{για } x=\frac{\pi}{2} \Rightarrow y=2\end{matrix}$ $= \int_1^2 \frac{1}{y} dy = \left[\ln y\right]_1^2 = \ln 2 - \ln 1 = \ln 2$

31.10 2)

$\int_0^{\frac{\pi}{2}} \eta\mu^2 x\sigma\upsilon\nu x\, dx$ $\begin{matrix}\text{θέτουμε } y=\eta\mu x \Rightarrow dy=\sigma\upsilon\nu x dx\\ \text{για } x=0 \Rightarrow y=0\\ \text{για } x=\frac{\pi}{2} \Rightarrow y=1\end{matrix}$ $= \int_0^1 y^2 dy = \left[\frac{y^3}{3}\right]_0^1 = \frac{1}{3}$

31.10 3)

$\int_0^{\pi} \sigma\upsilon\nu^5 x\eta\mu x\, dx$ $\begin{matrix}\text{θέτουμε } y=\sigma\upsilon\nu x \Rightarrow dy=-\eta\mu x dx\\ \text{για } x=0 \Rightarrow y=1\\ \text{για } x=\pi \Rightarrow y=-1\end{matrix}$ $= -\int_1^{-1} y^5 dy = \int_{-1}^1 y^5 dy = \left[\frac{y^6}{6}\right]_{-1}^1 = \frac{1^6}{6} - \frac{(-1)^6}{6} = 0$

31.10 4)

$\int_0^{\pi} \sigma\upsilon\nu^4 x \cdot \eta\mu x\, dx$ $\begin{matrix}\text{θέτουμε } y=\sigma\upsilon\nu x \Rightarrow dy=-\eta\mu x dx\\ \text{για } x=0 \Rightarrow y=1\\ \text{για } x=\pi \Rightarrow y=-1\end{matrix}$ $= -\int_1^{-1} y^4 dy = \int_{-1}^1 y^4 dy = \left[\frac{y^5}{5}\right]_{-1}^1 = \frac{1}{5} - \frac{-1}{5} = \frac{2}{5}$

31.10 5)

$\int_0^{\frac{\pi}{3}} \frac{\eta\mu x}{(1+2\sigma\upsilon\nu x)^2} dx$ $\begin{matrix}\text{θέτουμε } y=1+2\sigma\upsilon\nu x \Rightarrow dy=-2\eta\mu x dx \Rightarrow \eta\mu x dx=-\frac{dy}{2}\\ \text{για } x=0 \Rightarrow y=3\\ \text{για } x=\frac{\pi}{3} \Rightarrow y=2\end{matrix}$ $= -\int_3^2 \frac{1}{y^2}\frac{1}{2} dy = \frac{1}{2}\int_2^3 y^{-2} dy =$

$$= \frac{1}{2}\left[\frac{y^{-1}}{-1}\right]_2^3 = -\frac{1}{2}\left(\frac{1}{3}-\frac{1}{2}\right) = \frac{1}{12}$$

31.10 6)

$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{2\sigma\upsilon\nu^2 x\sqrt{\varepsilon\varphi x}} dx$ $\begin{matrix}\text{θέτουμε } y=\varepsilon\varphi x \Rightarrow dy=\frac{1}{\sigma\upsilon\nu^2 x} dx\\ \text{για } x=\frac{\pi}{4} \Rightarrow y=1\\ \text{για } x=\frac{\pi}{3} \Rightarrow y=\sqrt{3}\end{matrix}$ $= \int_1^{\sqrt{3}} \frac{1}{2\sqrt{y}} dy = \left[\sqrt{y}\right]_1^{\sqrt{3}} = \sqrt{\sqrt{3}} - 1 = \sqrt[4]{3} - 1$

31.10 7)

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{6}} \frac{1}{2\eta\mu^2 x\sqrt{\sigma\varphi x}}dx = \begin{matrix}\text{θέτουμε } y=\sigma\varphi x \Rightarrow dy=-\frac{1}{\eta\mu^2 x}dx \\ \text{για } x=\frac{\pi}{4}\Rightarrow y=1 \\ \text{για } x=\frac{\pi}{6}\Rightarrow y=\sqrt{3}\end{matrix} -\int_1^{\sqrt{3}} \frac{1}{2\sqrt{y}}dy = -\left[\sqrt{y}\right]_1^{\sqrt{3}} = -\sqrt{\sqrt{3}}+1 = 1-\sqrt[4]{3}$$

31.10 8)

$$\int_0^{\frac{\pi}{4}} \frac{\varepsilon\varphi x}{\sigma\upsilon\nu^2 x}dx = \begin{matrix}\text{θέτουμε } y=\varepsilon\varphi x \Rightarrow dy=\frac{1}{\sigma\upsilon\nu^2 x}dx \\ \text{για } x=0\Rightarrow y=0 \\ \text{για } x=\frac{\pi}{4}\Rightarrow y=1\end{matrix} \int_0^1 ydy = \left[\frac{y^2}{2}\right]_0^1 = \frac{1^2}{2}-\frac{0^2}{2} = \frac{1}{2}$$

31.10 9)

$$\int_{\frac{\pi}{2}}^{\frac{\pi}{4}} \frac{\sigma\varphi x}{\eta\mu^2 x}dx = \begin{matrix}\text{θέτουμε } y=\sigma\varphi x \Rightarrow dy=-\frac{1}{\eta\mu^2 x}dx \\ \text{για } x=\frac{\pi}{2}\Rightarrow y=0 \\ \text{για } x=\frac{\pi}{4}\Rightarrow y=1\end{matrix} -\int_0^1 ydy = -\left[\frac{y^2}{2}\right]_0^1 = -\frac{1^2}{2}+\frac{0^2}{2} = -\frac{1}{2}$$

31.10 10)

$$\int_0^{\sqrt{\frac{\pi}{6}}} 2x\sigma\upsilon\nu\left(x^2\right)dx = \begin{matrix}\text{θέτουμε } y=x^2 \Rightarrow dy=2xdx \\ \text{για } x=0\Rightarrow y=0 \\ \text{για } x=\sqrt{\frac{\pi}{6}}\Rightarrow y=\frac{\pi}{6}\end{matrix} \int_0^{\frac{\pi}{6}} \sigma\upsilon\nu ydy = \left[\eta\mu y\right]_0^{\frac{\pi}{6}} = \eta\mu\frac{\pi}{6}-\eta\mu 0 = \frac{1}{2}$$

31.10 11)

$$\int_0^{\frac{\pi^2}{36}} \frac{\sigma\upsilon\nu\sqrt{x}}{\sqrt{x}}dx = \begin{matrix}\text{θέτουμε } y=\sqrt{x} \Rightarrow dy=\frac{1}{2\sqrt{x}}dx \\ \Rightarrow \frac{1}{\sqrt{x}}dx=2dy \\ \text{για } x=0\Rightarrow y=0 \\ \text{για } x=\frac{\pi^2}{36}\Rightarrow y=\frac{\pi}{6}\end{matrix} \int_0^{\frac{\pi}{6}} 2\sigma\upsilon\nu ydy = 2\left[\eta\mu y\right]_0^{\frac{\pi}{6}} = 2\left(\eta\mu\frac{\pi}{6}-\eta\mu 0\right) = 1$$

31.10 12)

$$\int_{\log_2\frac{\pi-2}{2}}^{\log_2\frac{\pi-6}{6}} 2^x\sigma\upsilon\nu\left(2^x+1\right)dx = \begin{matrix}\text{θέτουμε } y=2^x+1 \Rightarrow dy=2^x\ln 2dx \\ \Rightarrow 2^x dx=\frac{1}{\ln 2}dy \\ \text{για } x=\log_2\frac{\pi-2}{2}\Rightarrow y=2^{\log_2\frac{\pi-2}{2}}+1\Rightarrow y=\frac{\pi}{2} \\ \text{για } x=\log_2\frac{\pi-6}{6}\Rightarrow y=2^{\log_2\frac{\pi-6}{6}}+1\Rightarrow y=\frac{\pi}{6}\end{matrix} \int_{\frac{\pi}{2}}^{\frac{\pi}{6}} \frac{1}{\ln 2}\sigma\upsilon\nu ydy =$$

$$= \frac{1}{\ln 2}\left[\eta\mu y\right]_{\frac{\pi}{2}}^{\frac{\pi}{6}} = \frac{1}{\ln 2}\left(\eta\mu\frac{\pi}{6}-\eta\mu\frac{\pi}{2}\right) = \frac{1}{\ln 2}\left(\frac{1}{2}-1\right) = -\frac{1}{2\ln 2}$$

31.10 13)

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\sigma\varphi x}{\eta\mu^2 x}dx = \begin{matrix}\text{θέτουμε } y=\sigma\varphi x \Rightarrow dy=-\frac{1}{\eta\mu^2 x}dx \\ \text{για } x=\frac{\pi}{6}\Rightarrow y=\sqrt{3} \\ \text{για } x=\frac{\pi}{2}\Rightarrow y=0\end{matrix} -\int_{\sqrt{3}}^0 ydy = -\left[\frac{y^2}{2}\right]_{\sqrt{3}}^0 = -\frac{0^2}{2}+\frac{\sqrt{3}^2}{2} = \frac{3}{2}$$

31.10 14)

$$\int_{\frac{3\pi}{2}}^{2\pi} \frac{\eta\mu x}{\left(1+\sigma\upsilon\nu x\right)^2}dx = \begin{matrix}\text{θέτουμε } y=1+\sigma\upsilon\nu x \Rightarrow dy=-\eta\mu x dx \\ \text{για } x=\frac{3\pi}{2}\Rightarrow y=1 \\ \text{για } x=2\pi\Rightarrow y=2\end{matrix} \int_1^2 -\frac{1}{y^2}dy = \left[\frac{1}{y}\right]_1^2 = \frac{1}{2}-\frac{1}{1} = -\frac{1}{2}$$

31.10 15)

$$\int_0^{\frac{\pi}{2}} \sigma\upsilon\nu x\sqrt{\eta\mu x}\,dx \overset{\substack{\theta\acute{\epsilon}\tau ουμε\ y=\eta\mu x\Rightarrow dy=\sigma\upsilon\nu x dx\\ \gamma\iota\alpha\ x=0\Rightarrow y=0\\ \gamma\iota\alpha\ x=\frac{\pi}{2}\Rightarrow y=1}}{=} \int_0^1 \sqrt{y}dy = \left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1 = \frac{1^{\frac{3}{2}}}{\frac{3}{2}} - \frac{0^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2}{3}$$

31.10 16)

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\eta\mu x\sigma\upsilon\nu x}{\sqrt{4\sigma\upsilon\nu^2 x+1}}dx \overset{\substack{\theta\acute{\epsilon}\tau ουμε\ y=4\sigma\upsilon\nu^2 x+1\Rightarrow dy=8\sigma\upsilon\nu x\eta\mu x dx\\ \eta\mu x\sigma\upsilon\nu x dx=\frac{dy}{8}\\ \gamma\iota\alpha\ x=\frac{\pi}{6}\Rightarrow y=4\\ \gamma\iota\alpha\ x=\frac{\pi}{2}\Rightarrow y=1}}{=} \int_1^4 \frac{1}{8}\frac{1}{\sqrt{y}}dy = \frac{2}{8}\left[\sqrt{y}\right]_1^4 = \frac{1}{4}(\sqrt{4}-\sqrt{1}) = \frac{1}{4}$$

31.10 17)

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\sqrt{\sigma\varphi x}}{\eta\mu^2 x}dx \overset{\substack{\theta\acute{\epsilon}\tau ουμε\ y=\sigma\varphi x\Rightarrow dy=-\frac{1}{\eta\mu^2 x}dx\\ \gamma\iota\alpha\ x=\frac{\pi}{4}\Rightarrow y=1\\ \gamma\iota\alpha\ x=\frac{\pi}{2}\Rightarrow y=0}}{=} -\int_1^0 \sqrt{y}dy = -\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^0 = -\frac{0^{\frac{3}{2}}}{\frac{3}{2}} + \frac{1^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2}{3}$$

31.10 18)

$$\int_0^{\frac{\pi}{3}} \frac{\varepsilon\varphi x}{\sigma\upsilon\nu x}dx = \int_0^{\frac{\pi}{3}} \frac{\frac{\eta\mu x}{\sigma\upsilon\nu x}}{\sigma\upsilon\nu x}dx = \int_0^{\frac{\pi}{3}} \frac{\eta\mu x}{\sigma\upsilon\nu^2 x}dx \overset{\substack{\theta\acute{\epsilon}\tau ουμε\ y=\sigma\upsilon\nu x\Rightarrow dy=-\eta\mu x dx\\ \gamma\iota\alpha\ x=0\Rightarrow y=1\\ \gamma\iota\alpha\ x=\frac{\pi}{3}\Rightarrow y=\frac{1}{2}}}{=} \int_1^{\frac{1}{2}} -\frac{1}{y^2}dy =$$

$$= \left[\frac{1}{y}\right]_1^{\frac{1}{2}} = \frac{1}{\frac{1}{2}} - \frac{1}{1} = 2-1 = 1$$