

$$\alpha) \int_0^{\frac{\pi}{2}} f(\eta\mu x) dx \stackrel{\substack{y=\frac{\pi}{2}-x \Rightarrow dy=-dx \\ x=\frac{\pi}{2}-y \\ \text{για } x=0 \Rightarrow y=\frac{\pi}{2} \\ \text{για } x=\frac{\pi}{2} \Rightarrow y=0}}{=} - \int_{\frac{\pi}{2}}^0 f\left[\eta\mu\left(\frac{\pi}{2}-y\right)\right] dy \stackrel{\eta\mu\left(\frac{\pi}{2}-y\right)=\sigma\upsilon\nu y}{=} \int_0^{\frac{\pi}{2}} f(\sigma\upsilon\nu y) dy =$$

$$= \int_0^{\frac{\pi}{2}} f(\sigma\upsilon\nu x) dx$$

$$\beta) \text{ Έστω } I = \int_0^{\frac{\pi}{2}} \frac{\sigma\upsilon\nu^2 x + \sigma\upsilon\nu x}{1 + \sigma\upsilon\nu x + \eta\mu x} dx \quad (1).$$

Έχουμε

$$I = \int_0^{\frac{\pi}{2}} \frac{\eta\mu^2 x + \eta\mu x}{1 + \eta\mu x + \sigma\upsilon\nu x} dx \stackrel{\substack{y=\frac{\pi}{2}-x \Rightarrow dy=-dx \\ x=\frac{\pi}{2}-y \\ \text{για } x=0 \Rightarrow y=\frac{\pi}{2} \\ \text{για } x=\frac{\pi}{2} \Rightarrow y=0}}{=} - \int_{\frac{\pi}{2}}^0 \frac{\eta\mu^2\left(\frac{\pi}{2}-y\right) + \eta\mu\left(\frac{\pi}{2}-y\right)}{1 + \eta\mu\left(\frac{\pi}{2}-y\right) + \sigma\upsilon\nu\left(\frac{\pi}{2}-y\right)} dy =$$

$$\stackrel{\substack{\eta\mu\left(\frac{\pi}{2}-y\right)=\sigma\upsilon\nu y \\ \sigma\upsilon\nu\left(\frac{\pi}{2}-y\right)=\eta\mu y}}{=} \int_0^{\frac{\pi}{2}} \frac{\sigma\upsilon\nu^2 y + \sigma\upsilon\nu y}{1 + \sigma\upsilon\nu y + \eta\mu y} dy = \int_0^{\frac{\pi}{2}} \frac{\sigma\upsilon\nu^2 x + \sigma\upsilon\nu x}{1 + \sigma\upsilon\nu x + \eta\mu x} dx$$

$$\text{Οπότε } I = \int_0^{\frac{\pi}{2}} \frac{\eta\mu^2 x + \eta\mu x}{1 + \eta\mu x + \sigma\upsilon\nu x} dx \quad (2)$$

Προσθέτουμε τις (1) και (2) κατά μέλη και έχουμε

$$2I = \int_0^{\frac{\pi}{2}} \frac{\eta\mu^2 x + \eta\mu x}{1 + \eta\mu x + \sigma\upsilon\nu x} dx + \int_0^{\frac{\pi}{2}} \frac{\sigma\upsilon\nu^2 x + \sigma\upsilon\nu x}{1 + \sigma\upsilon\nu x + \eta\mu x} dx \Rightarrow$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \frac{\eta\mu^2 x + \eta\mu x + \sigma\upsilon\nu^2 x + \sigma\upsilon\nu x}{1 + \eta\mu x + \sigma\upsilon\nu x} dx \Rightarrow 2I = \int_0^{\frac{\pi}{2}} \frac{1 + \eta\mu x + \sigma\upsilon\nu x}{1 + \eta\mu x + \sigma\upsilon\nu x} dx \Rightarrow$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 dx \Rightarrow 2I = 1 \left(\frac{\pi}{2} - 0 \right) \Rightarrow 2I = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$$