35.88

α) Έχουμε

$$I=\int_0^{\pi} x\cdot f(\eta\mu x)\,dx \overset{\substack{y=\pi-x\Rightarrow dy=-dx}}{\underset{\substack{\gamma\iota\alpha\ x=0\Rightarrow y=\pi\\ \gamma\iota\alpha\ x=\pi\Rightarrow y=0}}{=}} -\int_{\pi}^{0}(\pi-x)\cdot f(\eta\mu(\pi-x))\,dy=$$

$$=\int_0^{\pi}(\pi-y)\cdot f(\eta\mu y)\,dy=\int_0^{\pi}\pi\cdot f(\eta\mu y)\,dy-\int_0^{\pi}y\cdot f(\eta\mu y)\,dy=$$

$$=\int_0^{\pi}\pi\cdot f(\eta\mu x)\,dx-\int_0^{\pi}x\cdot f(\eta\mu x)\,dx=\pi\int_0^{\pi}f(\eta\mu x)\,dx-I$$

Άρα $I=\pi\int_0^{\pi}f(\eta\mu x)\,dx-I\Rightarrow 2I=\pi\int_0^{\pi}f(\eta\mu x)\,dx\Rightarrow I=\frac{\pi}{2}\int_0^{\pi}f(\eta\mu x)\,dx$

β) Έχουμε

$$\int_0^{\pi}\frac{x\eta\mu x}{3+\eta\mu^2x}dx=\int_0^{\pi}x\cdot\frac{\eta\mu x}{3+\eta\mu^2x}dx \overset{\alpha)\ \varepsilon\rho\acute{\omega}\tau\eta\mu\alpha}{=} \frac{\pi}{2}\int_0^{\pi}\frac{\eta\mu x}{3+\eta\mu^2x}dx \overset{\substack{y=\sigma\upsilon\nu x\Rightarrow dy=-\eta\mu x dx\\ \eta\mu^2x=1-\sigma\upsilon\nu^2x}}{\underset{\substack{\gamma\iota\alpha\ x=0\Rightarrow y=1\\ \gamma\iota\alpha\ x=\pi\Rightarrow y=-1}}{=}}$$

$$=\frac{\pi}{2}\int_1^{-1}-\frac{1}{4-y^2}dy=\frac{\pi}{2}\int_{-1}^{1}\frac{1}{4-y^2}dy$$

Άρα $\int_0^{\pi}\frac{x\eta\mu x}{3+\eta\mu^2x}dx=\frac{\pi}{2}\int_{-1}^{1}\frac{1}{4-y^2}dy \quad (1)$

Ακόμη

$$\frac{1}{4-y^2}=\frac{1}{(2-y)(2+y)}=\frac{A}{2-y}+\frac{B}{2+y}\Rightarrow 1=A(2+y)+B(2-y)\Rightarrow$$

$$\Rightarrow 1=2A+Ay+2B-By\Rightarrow 1=(A-B)y+(2A+2B)\Rightarrow\begin{cases}A-B=0\\ 2A+2B=1\end{cases}\Rightarrow$$

$$\Rightarrow A=B=\frac{1}{4}$$

Οπότε

$$(1)\Rightarrow\int_0^{\pi}\frac{x\eta\mu x}{3+\eta\mu^2x}dx=\frac{\pi}{2}\int_{-1}^{1}\frac{\frac{1}{4}}{2-y}+\frac{\frac{1}{4}}{2+y}dy=\frac{\pi}{8}\int_{-1}^{1}\frac{1}{2-y}+\frac{1}{2+y}dy=$$

$$=\frac{\pi}{8}\left[-\ln(2-y)+\ln(2+y)\right]_{-1}^{1}$$

$$=\frac{\pi}{8}\left[(-\ln1+\ln3)-(-\ln3+\ln1)\right]=\frac{\pi}{4}\ln3$$