Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Β

35.87

Έχουμε

$$\int_0^{\frac{\pi}{4}} \frac{1}{1+\eta\mu x}dx = \int_0^{\frac{\pi}{4}} \frac{1-\eta\mu x}{(1+\eta\mu x)(1-\eta\mu x)}dx = \int_0^{\frac{\pi}{4}} \frac{1-\eta\mu x}{1-\eta\mu^2 x}dx =$$

$$= \int_0^{\frac{\pi}{4}} \frac{1-\eta\mu x}{\sigma\upsilon\nu^2 x}dx = \int_0^{\frac{\pi}{4}} \frac{1}{\sigma\upsilon\nu^2 x}dx - \int_0^{\frac{\pi}{4}} \frac{\eta\mu x}{\sigma\upsilon\nu^2 x}dx$$

Άρα $\int_0^{\frac{\pi}{4}} \frac{1}{1+\eta\mu x}dx = \int_0^{\frac{\pi}{4}} \frac{1}{\sigma\upsilon\nu^2 x}dx - \int_0^{\frac{\pi}{4}} \frac{\eta\mu x}{\sigma\upsilon\nu^2 x}dx \quad (1)$

Ακόμη

$\int_0^{\frac{\pi}{4}} \frac{1}{\sigma\upsilon\nu^2 x}dx = [\varepsilon\varphi x]_0^{\frac{\pi}{4}} = \varepsilon\varphi\frac{\pi}{4} - \varepsilon\varphi 0 = 1$. Άρα $\int_0^{\frac{\pi}{4}} \frac{1}{\sigma\upsilon\nu^2 x}dx = 1 \quad (2)$

$$\int_0^{\frac{\pi}{4}} \frac{\eta\mu x}{\sigma\upsilon\nu^2 x}dx \underset{\substack{\gamma\iota\alpha\ x=0 \Rightarrow y=1 \\ \gamma\iota\alpha\ x=\frac{\pi}{4} \Rightarrow y=\frac{\sqrt{2}}{2}}}{\overset{y=\sigma\upsilon\nu x \Rightarrow dy=-\eta\mu x dx}{=}} \int_1^{\frac{\sqrt{2}}{2}} -\frac{1}{y^2}dy = \left[\frac{1}{y}\right]_1^{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}} - 1 = \sqrt{2} - 1$$

Άρα $\int_0^{\frac{\pi}{4}} \frac{\eta\mu x}{\sigma\upsilon\nu^2 x}dx = \sqrt{2} - 1 \quad (3)$

Οπότε

$$(1) \overset{(2),(3)}{\Rightarrow} \int_0^{\frac{\pi}{4}} \frac{1}{1+\eta\mu x}dx = 1 - \left(\sqrt{2} - 1\right) = 2 - \sqrt{2}$$