

$$\text{Είναι } f(x) + f(\alpha + \beta - x) = c \Rightarrow f(x) = c - f(\alpha + \beta - x) \quad (1)$$

Οπότε

$$\begin{aligned} \int_{\alpha}^{\beta} f(x) dx &\stackrel{(1)}{=} \int_{\alpha}^{\beta} [c - f(\alpha + \beta - x)] dx \quad \begin{array}{l} y = \alpha + \beta - x \Rightarrow dy = -dx \\ \text{για } x = \alpha \Rightarrow y = \beta \\ \text{για } x = \beta \Rightarrow y = \alpha \end{array} - \int_{\beta}^{\alpha} [c - f(y)] dy = \\ &= \int_{\alpha}^{\beta} [c - f(y)] dy = \int_{\alpha}^{\beta} c dy - \int_{\alpha}^{\beta} f(y) dy = c(\beta - \alpha) - \int_{\alpha}^{\beta} f(x) dx \end{aligned}$$

Άρα

$$2 \int_{\alpha}^{\beta} f(x) dx = c(\beta - \alpha) \Rightarrow \int_{\alpha}^{\beta} f(x) dx = \frac{c(\beta - \alpha)}{2} \quad (2)$$

Θα πρέπει τώρα να υπολογίσουμε το c . Είναι

$$f(x) + f(\alpha + \beta - x) = c \quad \begin{array}{l} \text{θέτουμε όπου } x \text{ το } \alpha \\ \Rightarrow \end{array} \quad c = f(\alpha) + f(\beta) \quad (3)$$

Ακόμη

$$\begin{aligned} f(x) + f(\alpha + \beta - x) = c &\quad \begin{array}{l} \text{θέτουμε όπου } x \text{ το } \frac{\alpha + \beta}{2} \\ \Rightarrow \end{array} \quad f\left(\frac{\alpha + \beta}{2}\right) + f\left(\frac{\alpha + \beta}{2}\right) = c \Rightarrow \\ \Rightarrow c &= 2f\left(\frac{\alpha + \beta}{2}\right) \quad (4) \end{aligned}$$

$$(2) \stackrel{(3), (4)}{\Rightarrow} \int_{\alpha}^{\beta} f(x) dx = (\beta - \alpha) f\left(\frac{\alpha + \beta}{2}\right) = \frac{\beta - \alpha}{2} [f(\alpha) + f(\beta)]$$