Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Β

35.82

Είναι

$$f'(x)=-e^{f(x)}-e^{-f(x)}+2 \Rightarrow e^{f(x)}f'(x)=-e^{2f(x)}-1+2e^{f(x)} \Rightarrow$$

$$\Rightarrow e^{f(x)}f'(x)=-\left(e^{2f(x)}+1-2e^{f(x)}\right) \Rightarrow e^{f(x)}f'(x)=-\left(e^{f(x)}-1\right)^2 \overset{e^{f(x)}-1=\frac{1}{g(x)}\neq 0}{\Rightarrow}$$

$$\Rightarrow -\frac{e^{f(x)}f'(x)}{\left(e^{f(x)}-1\right)^2}=1 \Rightarrow \left(\frac{1}{e^{f(x)}-1}\right)'=(x)' \overset{e^{f(x)}-1=\frac{1}{g(x)} \Rightarrow g(x)=\frac{1}{e^{f(x)}-1}}{\Rightarrow} g'(x)=(x)' \Rightarrow$$

$$\Rightarrow g(x)=x+c \ (1) \quad , \ c: \text{σταθερά}$$

$$(1) \overset{\text{θέτουμε } x=1}{\Rightarrow} g(1)=1+c \overset{g(1)=1}{\Rightarrow} c=0 \quad (2)$$

Οπότε

$$(1) \overset{(2)}{\Rightarrow} g(x)=x$$

Ακόμη

$$e^{f(x)}-1=\frac{1}{g(x)} \overset{g(x)=x}{\Rightarrow} e^{f(x)}-1=\frac{1}{x} \Rightarrow e^{f(x)}=\frac{1}{x}+1 \Rightarrow f(x)=\ln\left(\frac{1}{x}+1\right) \Rightarrow$$

$$\Rightarrow f(x)=\ln\frac{x+1}{x}$$