

α) Είναι $f'(x) = \frac{e^x}{\sqrt{x}}$ και $g'(x) = f(3x)$

Οπότε

$$g''(x) = f'(3x) \cdot (3x)' \Rightarrow \boxed{g''(x) = \frac{3e^{3x}}{\sqrt{3x}}} \Rightarrow g''(1) = \frac{3e^{3 \cdot 1}}{\sqrt{3 \cdot 1}} \Rightarrow$$

$$\Rightarrow g''(1) = \frac{3e^3}{\sqrt{3}} \Rightarrow g''(1) = \frac{\cancel{3}\sqrt{3}e^3}{\cancel{3}} \Rightarrow \boxed{g''(1) = e^3\sqrt{3}}$$

β) $\lim_{x \rightarrow 0^+} \frac{\sqrt{x}g''(x) - \sqrt{3}}{\sqrt{x+1} - 1} \stackrel{g''(x) = \frac{3e^{3x}}{\sqrt{3x}}}{=} \lim_{x \rightarrow 0^+} \frac{\sqrt{x} \frac{3e^{3x}}{\sqrt{3}\sqrt{x}} - \sqrt{3}}{\sqrt{x+1} - 1} = \lim_{x \rightarrow 0^+} \frac{\sqrt{3}e^{3x} - \sqrt{3}}{\sqrt{x+1} - 1} \stackrel{\frac{0}{0}}{=} \text{DLH}$

$$\lim_{x \rightarrow 0^+} \frac{3\sqrt{3}e^{3x} - 3\sqrt{3}}{2\sqrt{x+1} - 2} = 6\sqrt{3}e^3$$