

Επειδή η ευθεία $y = 2x + 1$, είναι πλάγια ασύμπτωτη στο $+\infty$, των γραφικών παραστάσεων των συναρτήσεων f και g σύμφωνα με γνωστή πρόταση θα ισχύει

$$\lim_{x \rightarrow +\infty} \frac{f(x)}{x} = 2 \quad (1) \quad , \quad \lim_{x \rightarrow +\infty} [f(x) - 2x] = 1 \quad (2)$$

$$\lim_{x \rightarrow +\infty} \frac{g(x)}{x} = 2 \quad (3) \quad , \quad \lim_{x \rightarrow +\infty} [g(x) - 2x] = 1 \quad (4)$$

Οπότε

$$(1) \quad \Rightarrow \quad \lim_{x \rightarrow +\infty} \frac{\frac{\alpha x^2 + \beta x + 2}{3x - 1}}{x} = 2 \Rightarrow \lim_{x \rightarrow +\infty} \frac{\alpha x^2 + \beta x + 2}{3x^2 - x} = 2 \Rightarrow$$

$$\Rightarrow \lim_{x \rightarrow +\infty} \frac{\alpha x^2}{3x^2} = 2 \Rightarrow \frac{\alpha}{3} = 2 \Rightarrow \alpha = 6$$

$$(2) \quad \Rightarrow \quad \lim_{x \rightarrow +\infty} \left[\frac{\frac{\alpha x^2 + \beta x + 2}{3x - 1}}{x} - 2 \right] = 1 \Rightarrow \lim_{x \rightarrow +\infty} \frac{\cancel{6x^2} + \beta x + 2 - \cancel{6x^2} + 2x}{3x - 1} = 1 \Rightarrow$$

$$\Rightarrow \lim_{x \rightarrow +\infty} \frac{(\beta + 2)x + 2}{3x - 1} = 1 \Rightarrow \lim_{x \rightarrow +\infty} \frac{(\beta + 2)x}{3x} = 1 \Rightarrow \frac{\beta + 2}{3} = 1 \Rightarrow \beta = 1$$

$$(3) \quad \Rightarrow \quad \lim_{x \rightarrow +\infty} \frac{\frac{g(x) = \sqrt{x^2 + 2x + 5} - \gamma x + \delta}{x}}{x} = 2 \Rightarrow$$

$$\Rightarrow \lim_{x \rightarrow +\infty} \frac{x \sqrt{1 + \frac{2}{x} + \frac{5}{x^2}} - \gamma x + \delta}{x} = 2 \Rightarrow \lim_{x \rightarrow +\infty} \frac{\cancel{x} \left(\sqrt{1 + \frac{2}{x} + \frac{5}{x^2}} - \gamma + \frac{\delta}{x} \right)}{\cancel{x}} = 2$$

$$\Rightarrow 1 - \gamma = 2 \Rightarrow \gamma = -1$$

$$(4) \quad \Rightarrow \quad \lim_{x \rightarrow +\infty} \left[\sqrt{x^2 + 2x + 5} + x + \delta - 2x \right] = 1 \Rightarrow$$

$$\Rightarrow \lim_{x \rightarrow +\infty} \left[\sqrt{x^2 + 2x + 5} - x + \delta \right] = 1 \Rightarrow \lim_{x \rightarrow +\infty} \left[\sqrt{x^2 + 2x + 5} - x \right] + \lim_{x \rightarrow +\infty} \delta = 1 \Rightarrow$$

$$\Rightarrow \lim_{x \rightarrow +\infty} \left[\frac{\sqrt{x^2 + 2x + 5}^2 - x^2}{\sqrt{x^2 + 2x + 5} + x} \right] + \delta = 1 \Rightarrow \lim_{x \rightarrow +\infty} \left[\frac{2x + 5}{x \sqrt{1 + \frac{2}{x} + \frac{5}{x^2}} + x} \right] + \delta = 1 \Rightarrow$$

$$\Rightarrow \lim_{x \rightarrow +\infty} \left[\frac{x \left(2 + \frac{5}{x} \right)}{x \left(\sqrt{1 + \frac{2}{x} + \frac{5}{x^2}} + 1 \right)} \right] + \delta = 1 \Rightarrow \frac{2+0}{\sqrt{1+0+0}+1} + \delta = 1 \Rightarrow 1 + \delta = 1 \Rightarrow$$

$$\Rightarrow \delta = 0$$