

Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Β

32.2 1)

$$\begin{aligned} E &= \int_{-6}^2 |f(x)| dx = \int_{-6}^2 |x| dx = \int_{-6}^0 |x| dx + \int_0^2 |x| dx = \int_{-6}^0 -x dx + \int_0^2 x dx = \\ &= -\left[\frac{x^2}{2}\right]_{-6}^0 + \left[\frac{x^2}{2}\right]_0^2 = -\left(\frac{0^2}{2} - \frac{(-6)^2}{2}\right) + \left(\frac{2^2}{2} - \frac{0^2}{2}\right) = 18 + 2 = \boxed{20} \end{aligned}$$

32.2 2)

$$\begin{aligned} E &= \int_2^6 |f(x)| dx = \int_2^6 |x-4| dx \stackrel{\substack{x-4>0 \Leftrightarrow x>4 \\ x-4<0 \Leftrightarrow x<4}}{=} \int_2^4 |x-4| dx + \int_4^6 |x-4| dx = \\ &= \int_2^4 (-x+4) dx + \int_4^6 (x-4) dx = \left[-\frac{x^2}{2} + 4x\right]_2^4 + \left[\frac{x^2}{2} - 4x\right]_4^6 = \\ &= -\frac{4^2}{2} + 4 \cdot 4 - \left(-\frac{2^2}{2} + 4 \cdot 2\right) + \frac{6^2}{2} - 4 \cdot 6 - \left(\frac{4^2}{2} - 4 \cdot 4\right) = \\ &= -8 + 16 - (-2 + 8) + 18 - 24 - (8 - 16) = 8 - 6 + 18 - 24 + 8 = 4 \end{aligned}$$

32.2 3)

$$\begin{aligned} E &= \int_{-1}^3 |f(x)| dx = \int_{-1}^3 |2x-4| dx \stackrel{\substack{2x-4>0 \Leftrightarrow x>2 \\ 2x-4<0 \Leftrightarrow x<2}}{=} \int_{-1}^2 |2x-4| dx + \int_2^3 |2x-4| dx = \\ &= \int_{-1}^2 (-2x+4) dx + \int_2^3 (2x-4) dx = \left[-x^2 + 4x\right]_{-1}^2 + \left[x^2 - 4x\right]_2^3 = \\ &= -2^2 + 4 \cdot 2 - \left(-(-1)^2 + 4(-1)\right) + 3^2 - 4 \cdot 3 - (2^2 - 4 \cdot 2) = \\ &= -4 + 8 - (-5) + 9 - 12 - (-4) = 10 \end{aligned}$$

32.2 4)

$$\begin{aligned} E &= \int_0^\pi |f(x)| dx = \int_0^\pi |\sin x| dx \stackrel{\substack{\sin x > 0 \Leftrightarrow 0 < x < \frac{\pi}{2} \\ \sin x < 0 \Leftrightarrow \frac{\pi}{2} < x < \pi}}{=} \int_0^{\frac{\pi}{2}} |\sin x| dx + \int_{\frac{\pi}{2}}^\pi |\sin x| dx = \\ &= \int_0^{\frac{\pi}{2}} \sin x dx + \int_{\frac{\pi}{2}}^\pi -\sin x dx = [\eta \mu x]_0^{\frac{\pi}{2}} + [-\eta \mu x]_{\frac{\pi}{2}}^\pi = \\ &= \eta \mu \frac{\pi}{2} - \eta \mu 0 - \eta \mu \pi + \eta \mu \frac{\pi}{2} = 2 \end{aligned}$$