30.31

$$f''(2x+1)=12x+6\Rightarrow 2\cdot f''(2x+1)=2\cdot(12x+6)\Rightarrow$$

$$\Rightarrow\left[f'(2x+1)\right]'=24x+12\Rightarrow\left[f'(2x+1)\right]'=\left(12x^2+12x\right)'\Rightarrow$$

$$\Rightarrow f'(2x+1)=12x^2+12x+c_1\overset{\substack{\text{θέτουμε } y=2x+1\\ \Rightarrow 2x=y-1\Rightarrow x=\frac{y-1}{2}}}{\Rightarrow} f'(y)=12\left(\frac{y-1}{2}\right)^2+12\frac{y-1}{2}+c_1\Rightarrow$$

$$f'(y)=\cancelto{3}{12}\frac{y^2-2y+1}{\cancel{4}}+\cancelto{6}{12}\frac{y-1}{\cancel{2}}+c_1\Rightarrow f'(y)=3\left(y^2-2y+1\right)+6(y-1)+c_1\Rightarrow$$

$$f'(y)=3y^2-\cancel{6y}+3+\cancel{6y}-6+c_1\Rightarrow f'(y)=3y^2-3+c_1\quad(1)$$

Όμως

$$f'(1)=4\Rightarrow 3\cdot1^2-3+c_1=4\Rightarrow c_1=4$$

Άρα

$$(1)\overset{c=4}{\Rightarrow}f'(y)=3y^2+1\Rightarrow f'(y)=\left(y^3+y\right)'\Rightarrow f(y)=y^3+y+c_2\quad(2)$$

Ακόμη

$$f(1)=1\overset{(1)}{\Rightarrow}1^3+1+c_2=1\Rightarrow c_2=-1$$

Άρα τελικά

$$(2)\overset{c=-1}{\Rightarrow}f(y)=y^3+y-1\ ,\quad y\in R$$