

Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Β

30.21 1)

$$f'(x^3) = 4x^3 + 1 \Rightarrow 3x^2 f'(x^3) = 3x^2 (4x^3 + 1) \Rightarrow$$

$$\left[f(x^3) \right]' = 12x^5 + 3x^2 \Rightarrow \left[f(x^3) \right]' = (2x^6 + x^3)' \Rightarrow f(x^3) = 2x^6 + x^3 + c \quad \text{θέτουμε } x^3 = y \Rightarrow$$

$$f(y) = 2y^2 + y + c \quad (1)$$

Όμως

$$f(1) = 4 \Rightarrow 2 \cdot 1^2 + 1 + c = 4 \Rightarrow c = 1$$

Άρα

$$(1) \Rightarrow f(y) = 2y^2 + y + 1, \quad y \in \mathbb{R}$$

30.21 2)

$$f'(x^2) = 3 - \frac{1}{x} \Rightarrow 2x f'(x^2) = 3 \cdot 2x - 2 \cancel{x} \frac{1}{\cancel{x}} \Rightarrow \left[f(x^2) \right]' = 6x - 2 \Rightarrow$$

$$\Rightarrow \left[f(x^2) \right]' = (3x^2 - 2x)' \Rightarrow f(x^2) = 3x^2 - 2x + c \quad \begin{matrix} \text{θέτουμε } y = x^2 \\ x > 0 \\ \Rightarrow x = \sqrt{y} \end{matrix} \Rightarrow f(y) = 3y - 2\sqrt{y} + c \quad (1)$$

Όμως

$$f(1) = 1 \Rightarrow 3 \cdot 1 - 2\sqrt{1} + c = 1 \Rightarrow c = 0$$

Άρα

$$(1) \Rightarrow f(y) = 3y - 2\sqrt{y}, \quad y > 0$$

30.21 3)

$$f'(\eta\mu x) = 3\eta\mu^2 x + 2 \Rightarrow \sigma\upsilon\nu x \cdot f'(\eta\mu x) = \sigma\upsilon\nu x \cdot (3\eta\mu^2 x + 2) \Rightarrow$$

$$\Rightarrow \left[f(\eta\mu x) \right]' = (\eta\mu^3 x + 2\eta\mu x)' \Rightarrow f(\eta\mu x) = \eta\mu^3 x + 2\eta\mu x + c \quad \text{θέτουμε } y = \eta\mu x \Rightarrow$$

$$\Rightarrow f(y) = y^3 + 2y + c \quad (1)$$

Όμως

$$f(1) = 3 \Rightarrow 1^3 + 2 \cdot 1 + c = 3 \Rightarrow c = 0$$

Άρα

$$(1) \Rightarrow f(y) = y^3 + 2y, \quad y \in [-1, 1]$$