

Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Β

30.20 1)

$$1 + F(x) = (x^2 + 1)e^x \xRightarrow{\text{παραγωγίζουμε}} F'(x) = 2xe^x + (x^2 + 1)e^x \xRightarrow{F'(x)=f'(x)}$$

$$\Rightarrow f(x) = 2xe^x + (x^2 + 1)e^x \Rightarrow f(x) = (x^2 + 1 + 2x)e^x \Rightarrow f(x) = (x+1)^2 e^x$$

30.20 2)

$$F(x) = f^2(x) \xRightarrow{\text{παραγωγίζουμε}} F'(x) = 2f(x)f'(x) \xRightarrow{F'(x)=\frac{f'(x)}{x}} \frac{f'(x)}{x} = 2f(x)f'(x) \xRightarrow{f(x) \neq 0}$$

$$\Rightarrow \frac{1}{x} = 2f'(x) \Rightarrow (\ln|x|)' = [2f(x)]' \xRightarrow{x>1 \Rightarrow x>0} 2f(x) = \ln|x| + c \Rightarrow f(x) = \frac{\ln x}{2} + \frac{c}{2} \quad (1)$$

Ακόμη

$$f(e^2) = 1 \xRightarrow{(1)} \frac{2}{2} + c = 1 \Rightarrow c = 0 \quad (2)$$

$$(1), (2) \Rightarrow f(x) = \frac{\ln x}{2}$$

30.20 3)

$$3 + F(x) = (2x^2 + x + 3)e^x \xRightarrow{\text{παραγωγίζουμε}} F'(x) = (4x + 1)e^x + (2x^2 + x + 3)e^x \Rightarrow$$

$$\Rightarrow F'(x) = (2x^2 + x + 3 + 4x + 1)e^x \xRightarrow{F'(x)=f'(x)} f'(x) = (2x^2 + 5x + 4)e^x$$

30.20 4)

$$F(x) = f^2(x) \xRightarrow{\text{παραγωγίζουμε}} F'(x) = 2f(x)f'(x) \xRightarrow{F'(x)=f'(x)} f'(x) = 2f(x)f'(x) \xRightarrow{f(x) \neq 0}$$

$$1 = 2f'(x) \Rightarrow f'(x) = \frac{1}{2} \Rightarrow f'(x) = \left(\frac{x}{2}\right)' \Rightarrow f(x) = \frac{x}{2} + c \quad (1)$$

Ακόμη

$$f(0) = 0 \xRightarrow{(1)} \frac{0}{2} + c = 0 \Rightarrow c = 0 \quad (2)$$

$$(1), (2) \Rightarrow f(x) = \frac{x}{2}$$

30.20 5)

$$e^x \sin x + F(x) = 1 + e^x f(x) \xRightarrow{\text{παραγωγίζουμε}} e^x \sin x - e^x \eta \mu x + F'(x) = e^x f(x) + e^x f'(x) \Rightarrow$$

$$\xRightarrow{F'(x)=xf'(x)} e^x \sin x - e^x \eta \mu x + \cancel{e^x f(x)} = \cancel{e^x f(x)} + e^x f'(x) \Rightarrow$$

$$\cancel{e^x} (\sin x - \eta \mu x) = \cancel{e^x} f'(x) \Rightarrow f'(x) = \sin x - \eta \mu x \Rightarrow f'(x) = (\eta \mu x + \sin x)' \Rightarrow$$

$$\Rightarrow f(x) = \eta \mu x + \sin x + c \quad (1)$$

Ακόμη

$$e^x \sin x + F(x) = 1 + e^x f(x) \xrightarrow{\text{θέτουμε } x=0} e^0 \sin 0 + F(0) = 1 + e^0 f(0) \Rightarrow$$

$$\begin{aligned} F(0)=0 \\ \Rightarrow 1 = 1 + f(0) \Rightarrow f(0) = 0 \end{aligned}$$

Οπότε

$$f(0) = 0 \xrightarrow{(1)} \eta \mu 0 + \sigma \nu 0 + c = 0 \Rightarrow c = -1 \quad (2)$$

$$(1), (2) \Rightarrow f(x) = \eta \mu x + \sigma \nu x - 1$$

30.20 6)

$$1 + F(x) = f(x) \xrightarrow{\text{παραγωγίζουμε}} F'(x) = f'(x) \Rightarrow f(x) = f'(x) \Rightarrow f(x) = ce^x \quad (1)$$

Ακόμη

$$1 + F(x) = f(x) \xrightarrow{\text{θέτουμε } x=0} 1 + F(0) = f(0) \xrightarrow{F(0)=0} f(0) = 1$$

Οπότε

$$f(0) = 1 \xrightarrow{(1)} ce^0 = 0 \Rightarrow c = 1 \quad (2)$$

$$(1), (2) \Rightarrow f(x) = e^x$$

30.20 7)

$$\begin{aligned} x + F(x) &= (x+1)f(x) \xrightarrow{\text{παραγωγίζουμε}} 1 + F'(x) = f(x) + (x+1)f'(x) \Rightarrow \\ F'(x)=f(x) \quad \Rightarrow \quad 1 + \cancel{f(x)} &= \cancel{f(x)} + (x+1)f'(x) \Rightarrow (x+1)f'(x) = 1 \xrightarrow{x>-1 \Rightarrow x+1 \neq 0} f'(x) = \frac{1}{x+1} \Rightarrow \end{aligned}$$

$$\Rightarrow f'(x) = [\ln|x+1|]' \xrightarrow{x>-1 \Rightarrow x+1>0} f(x) = \ln(x+1) + c \quad (1)$$

Ακόμη

$$x + F(x) = (x+1)f(x) \xrightarrow{\text{θέτουμε } x=0} 0 + F(0) = (0+1)f(0) \xrightarrow{F(0)=0} f(0) = 0$$

Οπότε

$$f(0) = 0 \xrightarrow{(1)} \ln(0+1) + c = 0 \Rightarrow c = 0 \quad (2)$$

$$(1), (2) \Rightarrow f(x) = \ln|x+1|$$

30.20 8)

$$\begin{aligned} xf(x) - F(x) &= \ln x \xrightarrow{\text{παραγωγίζουμε}} f(x) + xf'(x) - F'(x) = (\ln x)' \Rightarrow \\ F'(x)=f(x) \quad \Rightarrow \quad \cancel{f(x)} + xf'(x) - \cancel{f(x)} &= \frac{1}{x} \Rightarrow xf'(x) = \frac{1}{x} \xrightarrow{x \neq 0} f'(x) = \frac{1}{x^2} \Rightarrow \end{aligned}$$

$$\Rightarrow f'(x) = \left(-\frac{1}{x}\right)' \Rightarrow f(x) = -\frac{1}{x} + c \quad (1)$$

Ακόμη

$$xf(x) - F(x) = \ln x \quad \begin{matrix} \text{θέτουμε } x=1 \\ \Rightarrow \end{matrix} \quad 1f(1) - F(1) = \ln 1 \quad \begin{matrix} F(1)=0 \\ \Rightarrow \end{matrix} \quad f(1) = 0$$

Οπότε

$$f(1) = 0 \Rightarrow -\frac{1}{1} + c = 0 \Rightarrow c = 1 \quad (2)$$

$$(1), (2) \Rightarrow f(x) = -\frac{1}{x} + 1 \Rightarrow f(x) = \frac{x-1}{x}$$

30.20 9)

$$f(x) = F(x) \quad \begin{matrix} \text{παραγωγίζουμε} \\ \Rightarrow \end{matrix} \quad f'(x) = F'(x) \quad \begin{matrix} F'(x) = e^{x-f(x)} \\ \Rightarrow \end{matrix} \quad f'(x) = e^{x-f(x)} \Rightarrow f'(x) = \frac{e^x}{e^{f(x)}} \Rightarrow$$
$$\Rightarrow f'(x) e^{f(x)} = e^x \Rightarrow \left(e^{f(x)} \right)' = \left(e^x \right)' \Rightarrow e^{f(x)} = e^x + c \quad (1)$$

Ακόμη

$$f(x) = F(x) \quad \begin{matrix} \text{θέτουμε } x=0 \\ \Rightarrow \end{matrix} \quad f(0) = F(0) \quad \begin{matrix} F(0)=0 \\ \Rightarrow \end{matrix} \quad f(0) = 0$$

Οπότε

$$(1) \quad \begin{matrix} \text{θέτουμε } x=0 \\ \Rightarrow \end{matrix} \quad e^{f(0)} = e^0 + c \quad \begin{matrix} f(0)=0 \\ \Rightarrow \end{matrix} \quad c = 0 \quad (2)$$

$$(1), (2) \Rightarrow e^{f(x)} = e^x \Rightarrow f(x) = x$$