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$$\begin{aligned} (f(x))^2 + (g(x))^2 &= \eta\mu^2 x \stackrel{x=0}{\Rightarrow} f^2(0) + g^2(0) = \eta\mu^2 0 \Rightarrow f^2(0) + g^2(0) = 0 \Rightarrow \\ &\Rightarrow f(0) = 0 \text{ και } g(0) = 0 \end{aligned}$$

Ακόμη

$$\begin{aligned} f^2(x) &\leq f^2(x) + g^2(x) \stackrel{f^2(x)+g^2(x)=\eta\mu^2 x}{\Rightarrow} f^2(x) \leq \eta\mu^2 x \Rightarrow |f(x)| \leq |\eta\mu x| \Rightarrow \\ &\Rightarrow -|\eta\mu x| \leq f(x) \leq |\eta\mu x| \end{aligned}$$

Επομένως

$$\lim_{x \rightarrow 0} (-|\eta\mu x|) = \lim_{x \rightarrow 0} |\eta\mu x| = 0 \left\{ \begin{array}{l} \text{κριτήριο} \\ \text{παρεμβολής} \end{array} \right. \Rightarrow \lim_{x \rightarrow 0} f(x) = 0 \stackrel{f(0)=0}{\Rightarrow} \boxed{\lim_{x \rightarrow 0} f(x) = f(0)}$$

Οπότε η f είναι συνεχής στο $x_0 = 0$

Επίσης

$$\begin{aligned} g^2(x) &\leq f^2(x) + g^2(x) \stackrel{f^2(x)+g^2(x)=\eta\mu^2 x}{\Rightarrow} g^2(x) \leq \eta\mu^2 x \Rightarrow |g(x)| \leq |\eta\mu x| \Rightarrow \\ &\Rightarrow -|\eta\mu x| \leq g(x) \leq |\eta\mu x| \end{aligned}$$

Επομένως

$$\lim_{x \rightarrow 0} (-|\eta\mu x|) = \lim_{x \rightarrow 0} |\eta\mu x| = 0 \left\{ \begin{array}{l} \text{κριτήριο} \\ \text{παρεμβολής} \end{array} \right. \Rightarrow \lim_{x \rightarrow 0} g(x) = 0 \stackrel{g(0)=0}{\Rightarrow} \boxed{\lim_{x \rightarrow 0} g(x) = g(0)}$$

Οπότε και η g είναι συνεχής στο $x_0 = 0$