

Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Α

7.7 1)

$$\lim_{x \rightarrow +\infty} \frac{(1-x)(x^2+3)}{x^2+1} = \lim_{x \rightarrow +\infty} \frac{x \left(\frac{1}{x} - 1 \right) x^2 \left(1 + \frac{3}{x^2} \right)}{x^2 \left(1 + \frac{1}{x^2} \right)} = \frac{(+\infty)(0-1)(1+0)}{1+0} = (+\infty)(-1) = \boxed{-\infty}$$

7.7 2)

$$\lim_{x \rightarrow +\infty} \frac{x^2-5}{(x-2)(x+1)} = \lim_{x \rightarrow +\infty} \frac{x^2 \left(1 - \frac{5}{x^2} \right)}{x \left(1 - \frac{2}{x} \right) x \left(1 + \frac{1}{x} \right)} = \frac{1-0}{(1-0)(1+0)} = \boxed{1}$$

7.7 3)

$$\lim_{x \rightarrow +\infty} \frac{x \left(2 - \frac{3}{x} \right) x \left(3 + \frac{5}{x} \right) x \left(4 - \frac{6}{x} \right)}{x^3 \left(3 + \frac{1}{x^2} - \frac{1}{x^3} \right)} = \frac{(2-0)(3+0)(4-0)}{3+0-0} = \frac{24}{3} = \boxed{8}$$

7.7 4)

$$\begin{aligned} \lim_{x \rightarrow +\infty} \frac{(x-1)(2x-2)(x-3)(4x-4)(4x-5)}{(2x-1)^5} &= \\ &= \lim_{x \rightarrow +\infty} \frac{x \left(1 - \frac{1}{x} \right) x \left(2 - \frac{2}{x} \right) x \left(1 - \frac{3}{x} \right) x \left(4 - \frac{4}{x} \right) x \left(4 - \frac{5}{x} \right)}{x^5 \left(2 - \frac{1}{x} \right)^5} = \\ &= \frac{(1-0)(2-0)(1-0)(4-0)(4-0)}{(2-0)^5} = \frac{32}{32} = \boxed{1} \end{aligned}$$

7.7 5)

$$\lim_{x \rightarrow +\infty} \frac{(3x-1)^3 (2x+1)^4}{x^7+2} = \lim_{x \rightarrow +\infty} \frac{x^3 \left(3 - \frac{1}{x} \right)^3 x^4 \left(1 + \frac{1}{x} \right)^4}{x^7 \left(1 + \frac{2}{x^7} \right)} = \frac{(3-0)^3 (1+0)^4}{1+0} = 3^3 \cdot 1^4 = \boxed{27}$$

7.7 6)

$$\begin{aligned} \lim_{x \rightarrow +\infty} \frac{(x-3)^{37} (4x+1)^{13}}{(2x^2-2)^{25}} &= \lim_{x \rightarrow +\infty} \frac{x^{37} \left(1 - \frac{3}{x} \right)^{37} x^{13} \left(4 + \frac{1}{x} \right)^{13}}{(x^2)^{25} \left(2 - \frac{2}{x^2} \right)^{25}} = \\ &= \lim_{x \rightarrow +\infty} \frac{x^{50} \left(1 - \frac{3}{x} \right)^{37} \left(4 + \frac{1}{x} \right)^{13}}{x^{50} \left(2 - \frac{2}{x^2} \right)^{25}} = \frac{(1-0)^{37} (4+0)^{13}}{(2-0)^{25}} = \frac{4^{13}}{2^{25}} = \frac{2^{26}}{2^{25}} = \boxed{2} \end{aligned}$$