

# ΓΛΥΚΕΙΟΥ ΜΕΡΟΣ Α

**7.31**

$$\begin{aligned}
 & \lim_{x \rightarrow +\infty} \frac{\left(x - \sqrt{x^2 - 1}\right)^{40} + \left(x + \sqrt{x^2 + 1}\right)^{40}}{x^{40}} = \\
 &= \lim_{x \rightarrow +\infty} \frac{\left(x - \sqrt{x^2} \sqrt{1 - \frac{1}{x^2}}\right)^{40} + \left(x + \sqrt{x^2} \sqrt{1 + \frac{1}{x^2}}\right)^{40}}{x^{40}} = \\
 &= \lim_{x \rightarrow +\infty} \frac{\left(x - |x| \sqrt{1 - \frac{1}{x^2}}\right)^{40} + \left(x + |x| \sqrt{1 + \frac{1}{x^2}}\right)^{40}}{x^{40}} \underset{|x|=x}{=} \\
 &= \lim_{x \rightarrow +\infty} \frac{\left(x - x \sqrt{1 - \frac{1}{x^2}}\right)^{40} + \left(x + x \sqrt{1 + \frac{1}{x^2}}\right)^{40}}{x^{40}} = \\
 &= \lim_{x \rightarrow +\infty} \frac{x^{40} \left(1 - \sqrt{1 - \frac{1}{x^2}}\right)^{40} + x^{40} \left(1 + \sqrt{1 + \frac{1}{x^2}}\right)^{40}}{x^{40}} = \\
 &= \lim_{x \rightarrow +\infty} \frac{x^{40} \left[ \left(1 - \sqrt{1 - \frac{1}{x^2}}\right)^{40} + \left(1 + \sqrt{1 + \frac{1}{x^2}}\right)^{40} \right]}{x^{40}} = \left(1 - \sqrt{1 - 0}\right)^{40} + \left(1 + \sqrt{1 + 0}\right)^{40} = \boxed{2^{40}}
 \end{aligned}$$