

7.20 1)

$$\begin{aligned} \lim_{x \rightarrow +\infty} \left(x^2 \eta\mu \frac{1}{x} \right) & \stackrel{\substack{\text{θέτουμε } y = \frac{1}{x} \Rightarrow x = \frac{1}{y} \\ \text{όταν } x \rightarrow +\infty \text{ τότε } y \rightarrow 0^+}}{=} \lim_{x \rightarrow +\infty} \left(\frac{1}{y^2} \eta\mu y \right) = \\ & = \lim_{x \rightarrow 0^+} \left(\frac{\eta\mu y}{y} \cdot \frac{1}{y} \right) \stackrel{\substack{\lim_{x \rightarrow 0^+} \frac{\eta\mu y}{y} = 1, \lim_{x \rightarrow 0^+} \frac{1}{y} = +\infty}}{=} 1 \cdot (+\infty) = \boxed{+\infty} \end{aligned}$$

7.20 2)

$$\lim_{x \rightarrow +\infty} \left(\eta\mu \frac{1}{x} \right) \stackrel{\substack{\text{θέτουμε } y = \frac{1}{x} \\ \text{όταν } x \rightarrow +\infty \text{ τότε } y \rightarrow 0^+}}{=} \lim_{y \rightarrow 0^+} (\eta\mu y) = \boxed{0}$$

7.20 3)

$$\lim_{x \rightarrow +\infty} \left(x \eta\mu \frac{1}{x} \right) \stackrel{\substack{\text{θέτουμε } y = \frac{1}{x} \Rightarrow x = \frac{1}{y} \\ \text{όταν } x \rightarrow +\infty \text{ τότε } y \rightarrow 0^+}}{=} \lim_{y \rightarrow 0^+} \left(\frac{1}{y} \eta\mu y \right) \stackrel{\lim_{x \rightarrow 0^+} \frac{\eta\mu y}{y} = 1}{=} \boxed{1}$$

7.20 4)

$$\begin{aligned} \lim_{x \rightarrow -\infty} \left(x^2 \varepsilon\phi \frac{1}{x} \right) & \stackrel{\substack{\text{θέτουμε } y = \frac{1}{x} \Rightarrow x = \frac{1}{y} \\ \text{όταν } x \rightarrow -\infty \text{ τότε } y \rightarrow 0^-}}{=} \lim_{x \rightarrow -\infty} \left(\frac{1}{y^2} \varepsilon\phi y \right) = \lim_{x \rightarrow -\infty} \left(\frac{1}{y^2} \frac{\eta\mu y}{\sigma\upsilon\nu y} \right) = \\ & = \lim_{x \rightarrow -\infty} \left(\frac{1}{\sigma\upsilon\nu y} \cdot \frac{1}{y} \cdot \frac{\eta\mu y}{y} \right) \stackrel{\substack{\lim_{x \rightarrow 0^-} \frac{\eta\mu y}{y} = 1, \lim_{x \rightarrow 0^-} \frac{1}{y} = -\infty}}{=} \frac{1}{1} \cdot (-\infty) \cdot 1 = \boxed{-\infty} \end{aligned}$$

7.20 5)

$$\lim_{x \rightarrow -\infty} \left(x^2 \sigma\upsilon\nu \frac{1}{x} \right) \stackrel{\substack{\text{θέτουμε } y = \frac{1}{x} \Rightarrow x = \frac{1}{y} \\ \text{όταν } x \rightarrow -\infty \text{ τότε } y \rightarrow 0^-}}{=} \lim_{y \rightarrow 0^-} \left(\frac{1}{y^2} \sigma\upsilon\nu y \right) = (+\infty) \cdot 1 = \boxed{+\infty}$$