

Γ ΛΥΚΕΙΟΥ ΜΕΡΟΣ Α

7.2 1)

$$\begin{aligned}\alpha) \quad \lim_{x \rightarrow -\infty} (2x^2 + x - 1)(x^2 - 3x + 9) &= \lim_{x \rightarrow -\infty} x^2 \left(2 + \frac{1}{x} - \frac{1}{x^2} \right) x^2 \left(1 - \frac{3}{x} + \frac{9}{x^2} \right) = \\ &= \lim_{x \rightarrow -\infty} x^4 \left(2 + \frac{1}{x} - \frac{1}{x^2} \right) \left(1 - \frac{3}{x} + \frac{9}{x^2} \right) = (-\infty)(2+0-0)(1-0+0) = \boxed{-\infty} \\ \beta) \quad \lim_{x \rightarrow -\infty} (x^3 - x - 1)^3 &= \lim_{x \rightarrow -\infty} \left[x^3 \left(1 - \frac{1}{x^2} - \frac{1}{x^3} \right) \right]^3 = \lim_{x \rightarrow -\infty} x^9 \left(1 - \frac{1}{x^2} - \frac{1}{x^3} \right)^3 = \\ &= \lim_{x \rightarrow -\infty} x^9 \left(1 - \frac{1}{x^2} - \frac{1}{x^3} \right)^3 = (-\infty)(1-0-0)^3 = \boxed{-\infty}\end{aligned}$$

7.2 2)

$$\begin{aligned}\lim_{x \rightarrow +\infty} (x+3)(x-4) &= \lim_{x \rightarrow +\infty} x \left(1 + \frac{3}{x} \right) x \left(1 - \frac{4}{x} \right) = \lim_{x \rightarrow +\infty} x^2 \left(1 + \frac{3}{x} \right) \left(1 - \frac{4}{x} \right) = \\ &= (+\infty)(1+0)(1-0) = \boxed{+\infty}\end{aligned}$$

7.2 3)

$$\begin{aligned}\lim_{x \rightarrow -\infty} (x^2 + x)(x^3 - 2x + 1) &= \lim_{x \rightarrow -\infty} x^2 \left(1 + \frac{1}{x} \right) x^3 \left(1 - \frac{2}{x^2} + \frac{1}{x^3} \right) = \\ &= \lim_{x \rightarrow -\infty} x^5 \left(1 + \frac{1}{x} \right) \left(1 - \frac{2}{x^2} + \frac{1}{x^3} \right) = (-\infty)(1+0)(1-0+0) = \boxed{-\infty}\end{aligned}$$

7.2 4)

$$\begin{aligned}\lim_{x \rightarrow -\infty} (x^2 + 5x)(x^2 - 2x)(x - 2) &= \lim_{x \rightarrow -\infty} x^2 \left(1 + \frac{5}{x} \right) x^2 \left(1 - \frac{2}{x} \right) x \left(1 - \frac{2}{x} \right) = \\ &= \lim_{x \rightarrow -\infty} x^5 \left(1 + \frac{5}{x} \right) \left(1 - \frac{2}{x} \right) \left(1 - \frac{2}{x} \right) = (-\infty)(1+0)(1-0)(1-0) = \boxed{-\infty}\end{aligned}$$

7.2 5)

$$\begin{aligned}\lim_{x \rightarrow +\infty} (x+1)(x-7)(x-9)(-2x+1) &= \lim_{x \rightarrow +\infty} x \left(1 + \frac{1}{x} \right) x \left(1 - \frac{7}{x} \right) x \left(1 - \frac{9}{x} \right) x \left(-2 + \frac{1}{x} \right) = \\ &= \lim_{x \rightarrow +\infty} x^4 \left(1 + \frac{1}{x} \right) \left(1 - \frac{7}{x} \right) \left(1 - \frac{9}{x} \right) \left(-2 + \frac{1}{x} \right) = (+\infty)(1+0)(1-0)(1-0)(-2+0) = \boxed{-\infty}\end{aligned}$$

7.2 6)

$$\begin{aligned}\lim_{x \rightarrow +\infty} (2x^2 + x + 3)^2 &= \lim_{x \rightarrow +\infty} \left[x^2 \left(2 + \frac{1}{x} + \frac{3}{x^2} \right) \right]^2 = \lim_{x \rightarrow +\infty} x^4 \left(2 + \frac{1}{x} + \frac{3}{x^2} \right)^2 = \\ &= (+\infty)(2+0+0)^2 = \boxed{+\infty}\end{aligned}$$

7.2 7)

$$\begin{aligned}\lim_{x \rightarrow -\infty} (-x^3 + x + 3)^5 &= \lim_{x \rightarrow -\infty} \left[x^3 \left(-1 + \frac{1}{x^2} + \frac{3}{x^3} \right) \right]^5 = \lim_{x \rightarrow -\infty} x^{15} \left(-1 + \frac{1}{x^2} + \frac{3}{x^3} \right)^5 = \\ &= (-\infty)(-1+0+0)^5 = \boxed{+\infty}\end{aligned}$$

